Computers and Structures 193 (2017) 155–171

Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

An enhanced numerical procedure for the shakedown analysis in multidimensional loading domains

K.V. Spiliopoulos [a,*], K.D. Panagiotou [b,c]

[a] Institute of Structural Analysis & Antiseismic Research, Department of Civil Engineering, National Technical University of Athens, Zografou Campus, 157-80 Athens, Greece
[b] AICES, RWTH Aachen University, Schinkelstraße 2, 52062 Aachen, Germany
[c] Institute of Applied Mechanics, RWTH Aachen University, Mies-van-der-Rohe-Straße 1, 52074 Aachen, Germany

ARTICLE INFO

Article history:
Received 6 June 2017
Accepted 15 August 2017

Keywords:
Numerical algorithms
Direct methods
Cyclic loading
Plasticity
Shakedown
Residual stresses

ABSTRACT

The Residual Stress Decomposition Method for Shakedown (RSDM-S) is a new iterative direct method to estimate the shakedown load in a 2-dimensional (2D) loading domain. It may be implemented to any existing finite element code, without the need to use a mathematical programming algorithm. An improved and enhanced RSDM-S is proposed herein. A new convergence criterion is presented that makes the procedure almost double as fast. At the same time, the procedure is formulated in a 3-dimensional (3D) polyhedral loading domain, consisting of independently varying mechanical and thermal loads. Using a cyclic loading program that follows the outline of this domain, it is shown that there is hardly any increase in the computational time when passing from a 2D to a 3D domain. Finally, keeping the efficiency, using an alternative cyclic loading program, an automation of the approach to any n-dimensional loading domain is presented. Examples of application are included.

1. Introduction

A major task in civil and mechanical engineering is the estimation of the load carrying capacity of a structure or a component under variable loadings. Structures, like buildings, bridges, pavements, nuclear reactors, aircraft propulsion engines, etc., during their lifetime, are subjected to loads (live load, heavy traffic, seismic action, internal pressure, thermo-mechanical loads, etc.) acting in a varying manner. This type of cyclic mechanical and thermal loading leads often these structures beyond the elastic limit, resulting to plastic straining.

The asymptotic cyclic behavior of an elastic-perfectly plastic structure under cyclic loading may be determined by time consuming incremental time-stepping calculations. Direct methods, alternatively, have a big computational advantage as they attempt to find directly this cyclic asymptotic state. Such states are guaranteed for structures made of stable material [1].

There are a few direct methods, proposed in the literature, among which one may mention the work presented in [2,3] which forms a sequence of elastic solutions using as a modified loading an update of initial strains computed through an update of internal variables. This method is the basis of a recently presented direct method [4]. Approaches based also on a series of elastic analyses produced by modifying, iteratively, the modulus of elasticity, form another class of direct methods. Among them one should mention the Linear Matching Method (LMM) [5,6]. An incremental-iterative procedure, that appears to work well in cases of alternative plasticity but not for cases of ratcheting, was proposed in [7] and has been implemented in a commercial code. Very recently, a numerical scheme is presented, based on the conditions of the asymptotic state linked with a specific trial and projection operation, to estimate the plastic strain increments [8].

A direct method, which is known as the Residual Stress Decomposition Method (RSDM), was presented in [9,10]. The method can predict the long-term cyclic state, either it is shakedown or reverse plasticity or incremental collapse, of an elastic perfectly-plastic structure when subjected to a given cyclic loading history. The approach is based on physical arguments that have to do with the expected cyclic nature of the residual stresses. The residual stresses are decomposed into Fourier series with respect to time and the coefficients of these series are calculated iteratively by satisfying equilibrium and compatibility at time points inside the cycle.

When, on the other hand, the loading history is unknown, for a structure to be safe and serviceable, safety margins, e.g. shakedown limits, have to be estimated so that it fails neither due to incremental collapse (often referred to as ratcheting) nor due to reverse plasticity that leads to low cycle fatigue. A direct

* Corresponding author.
E-mail address: kvspilio@central.ntua.gr (K.V. Spiliopoulos).

http://dx.doi.org/10.1016/j.compstruc.2017.08.008
0045-7949/

shakedown analysis is the only way to provide this information. For small displacements and elastic-perfectly plastic solids the shakedown analysis is based on two different approaches, the lower bound [11] or the upper bound [12] shakedown theorems. The extensions of these two theorems to cover thermal loadings were given in [13,14], respectively.

Attempts to consider geometric nonlinearities appeared in the literature (e.g. [15,16]). Conditions to extend the static theorem to elastic-perfectly plastic cracked bodies have been presented in [17]. Limited kinematic (e.g. [18]) and nonlinear kinematic hardening has been also addressed (e.g. [19]). Recent developments on the subject have appeared in [20,21] in the framework of the bipotential theory. Non-associated plasticity has also been discussed (e.g. [22,23]). Polizzotto, has discussed the shakedown theorems in the context of gradient plasticity theory [24,25].

The two shakedown theorems form the basis of the big majority of the existing numerical procedures to estimate the shakedown load. They are formulated as mathematical programming (MP) problems whose scope is to find the minimum or maximum value of an objective function (normally the loading factor) which is subjected to various static or kinematic constraints. Linearization, mainly of the yield surface, has led to some early solutions using linear programming algorithms (e.g. [26,27]). More recent contributions have appeared along the same line (e.g. [28–30]). If the constraints are not linearized and are kept in their original form (nonlinear), the problem can be formulated as a nonlinear (NLP) programming problem. The discretization of the continuum by a large number of finite elements and the big number of constraints often lead to the solution of large size optimization problems. Various numerical techniques have been developed to solve these problems. Among these one could mention the reduced basis technique [19,31] or algorithms based on Newton iterations [32]. The evolution of the interior point algorithms (IPM) to solve large scale optimization problems led to the extensive formulation and solution of limit and shakedown analysis problems using these algorithms or related techniques (e.g. [33–43]).

One may also find some alternative approaches in the literature for the evaluation of the shakedown load. Such an approach is based on the work presented in [2], whose application using the finite element method (FEM) may be found in subsequent publications (e.g. [44]). Reverse plasticity and collapse load solutions have been shown to provide upper bounds to the shakedown load [45]. The LMM has also been used to estimate the shakedown load of a structure (e.g. [5,46]). In [47] a solution is proposed, based on the LMM, to estimate a possible shakedown load when friction slip occurs between a rigid surface in contact with an elastic body, subjected to cyclic loading. A quite involved strategy, equivalent to a fictitious incremental strain driven elastoplastic problem, and applied for a von Mises type of material, has been presented in [48]. The numerical performance of this approach is compared against the IPMs in [49]. An analogous methodology, involving more general yield criteria, was proposed in [50].

A numerical approach, which was called RSDM-S has appeared recently [51–53]. It may be used for the evaluation of the shakedown load of elastic-perfectly plastic structural elements under cyclic thermo-mechanical loading. The basis of the method, both from the conceptual as well as the implementation points of view, is the RSDM. Since, now, only the variation intervals of the loads are known, the problem is converted to an equivalent prescribed loading problem, drawing any time curve crossing these intervals. The RSDM-S consists of two different iteration loops, one inside the other and has been formulated for two loads that may vary either proportionally or independently. Starting from a high load factor, a descending sequence of loading factors is established and the shakedown load factor is calculated when the iterative procedure converges to a solution where the constant term is the only non-zero term of the Fourier series.

The efficiency of the RSDM-S and RSDM to provide shakedown boundaries as well as to unveil unsafe conditions in 2-dimensional loading domains was recently demonstrated in [54].

In the present work, the RSDM-S method is enhanced by a different convergence criterion, inside the inner loop, that makes the method run faster, even more than 40%. Moreover, the method is formulated for a 3-dimensional loading domain consisting of two mechanical and a thermal load. It is shown that the extension from a 2-D to a 3-D loading domain hardly influences the amount of computational time to estimate the shakedown load factor as opposed to the IPM algorithms where the time is shown to double [55]. Finally, it is shown how the method may be automated to cater for any n-dimensional loading domain.

The paper is organized in the following way: In Section 2 a proof of an existing theorem makes possible to realize the arbitrariness of the cyclic loading program that passes through the vertices of the convex loading domain; in Section 3 the enhanced RSDM-S procedure, in the form of a flow chart, with the new convergence criterion, formulated in a 3-D thermomechanical loading domain and assuming a von Mises yield criterion, is presented. The significant faster convergence of the enhanced approach is demonstrated through examples of 2-D loading domains in Section 4. In Section 5 the method is applied to a 3-D polyhedral loading domain using a cyclic loading program that passes consecutively from all its vertices. Finally, in Section 6 an alternative cyclic loading path combined with a combinatorial algorithm shows how the whole procedure may be automated for an n-dimensional domain.

2. Theoretical considerations

Let us suppose a structure is subjected to independently varying cyclic loads that have the same period T. Although the theory may be applied to any number of loads, for reasons of visualization, a maximum of three loading (3-D) domain that consists of two mechanical and a thermal load will be demonstrated (Fig. 1(a)). Such a cyclic loading may be represented in the loading space as a closed loop (Fig. 1(b)). Let us further suppose that each load has a minimum and a maximum value of variation. Without any loss of generality, the minimum of all the loads will be considered zero. The maximum of each of the loads, denoted by starred quantities, together with the origin may define a convex (hyper-) cuboid (Fig. 1(b)). Thus, the cyclic loading will be contained inside this cuboid.

In response to this loading the structure that consists of an elastic-perfectly plastic material will develop a stress that may be decomposed into two parts; an elastic part assuming purely elastic material behavior and a residual stress part to account for plasticity:

$$\boldsymbol{\sigma}(\tau) = \boldsymbol{\sigma}^{el}(\tau) + \boldsymbol{\rho}(\tau) \tag{1}$$

where $\tau = t/T$ denotes a time point inside the cycle.

The structure is discretized, following a standard procedure, into a finite number of elements that are interconnected at a discrete number of nodal points situated on their boundaries. Bold letters are herein used for vectors and matrices. The stress and strain vectors are evaluated at the Gauss points (GPs) of the finite elements (FE).

The strain rates, on the other hand, may be decomposed into the following parts:

$$\dot{\boldsymbol{\varepsilon}}(\tau) = \dot{\mathbf{e}}^{el}(\tau) + \dot{\mathbf{e}}^{\theta}(\tau) + \dot{\boldsymbol{\varepsilon}}_r^{el}(\tau) + \dot{\boldsymbol{\varepsilon}}^{pl}(\tau) \tag{2}$$

where $\dot{\mathbf{e}}^{el}(\tau)$ is the elastic straining due to both the mechanical and the thermal loading [52]. $\dot{\mathbf{e}}^{\theta}(\tau)$ denotes thermal strain rates that