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The influence of damage on the eigen-properties of Timoshenko spatial arches

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ABSTRACT

This paper studies the influence of concentrated damage on the eigen-properties of Timoshenko curved beams. Either single spatial arches or frame dome structures, obtained as assemblage of circular Timoshenko arches, are considered in presence of single or multiple damage and their exact dynamic stiffness matrices are evaluated. The natural frequencies and the corresponding modes of vibration are exactly calculated by means of a numerical strategy based on the Wittrick & Williams algorithm. The proposed procedure allows evaluating the effects of damage positions and intensities on the eigen-properties of the considered arch structures and to observe that, in the case of arches, the effects of damage severity and its location cannot be rigorously uncoupled. This latter result appears to be in contrast to what obtained in the literature for beams and rods. Since the adopted numerical approach leads to exact solutions, the obtained results can also be used as benchmarks for validating approximate numerical FEM strategies of Timoshenko damaged curved beams.

1. Introduction

In civil and mechanical engineering it is often adopted the design of frame structures composed by straight and curved bars which may be either plane or spatial. The free vibration analysis of these structures has been widely investigated in the last century because of its great practical importance in the study of the dynamic behavior of various structures such as bridges, vaults, underground galleries, and components of mechanical engineering.

A great number of papers on the subject have been published in the scientific literature, the largest part of these publications deal with circular arches [1–6], nevertheless some studies have been presented concerning the vibration of arches with variable curvature [7–9]. A fundamental review has been published, in 1993, by Chidamparam and Leissa taking into account more than 400 references [10].

In the free vibration of curved beams either in-plane or out-of-plane motions may occur, where the former consist primarily of bending-extensional modes while the latter is essentially bending-twisting dynamics [3,4]. In order to study the lowest natural frequency, which is predominantly flexural, many researchers have formulated the hypothesis of inextensibility of the centreline of the arch either in studying arches with constant or variable curvature. Some authors also investigated the influence of centreline extensibility on the free vibrations of arches [11], other effects, such as rotatory inertia and shear effects have been also studied [12].

The presence of damage in beam-like structures, either straight or curve, implies a loss of the structural stiffness inducing variation of both static and dynamic responses [13–15]. The effects of damage have been investigated in the literature, mainly with reference to straight beams, using different techniques. These are based on the variation of dynamic characteristics, such as natural frequencies, mode shapes, dynamic flexibilities, or static quantities, such as displacement induced by applied loads [16–19]. The most interesting application of these techniques can be found in the health monitoring strategy, whereby the using of experimental data from non-destructive tests allows to identify damage [20–22]. A recent overview on damage identification techniques via modal curvature analysis has been given by Dessi and Camerlengo [23].

In this paper, spatial arch structures, in which the damage is modeled as a reduction of the cross section in a concentrated zone [24], are studied in their undamaged and damaged states.

The exact natural frequencies of vibration and the corresponding modes are calculated by means of an application of the Wittrick & Williams algorithm [25] by following a procedure introduced by Howson et al. [3,4] and revisited by the authors in a previous paper

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[26] in which the dynamic of undamaged spatial arch structures has been exactly investigated in the context of the dynamic stiffness method [27–29].

By studying the direct problem, the sensitivity of the modal properties to damage can be assessed by considering the three parameters assumed as characteristics of a single damage, i.e. location, length of the damaged zone and amount of reduction of the area of the cross section. An extensive parametric study is performed showing the effect of the damage parameters on the natural frequencies of vibration of a single spatial arch. The complex case of the presence of multiple damage, on the considered structure, is also briefly analysed with reference to some damage scenarios accounting for variability of positions and severity of damage.

Some peculiar physical properties, obtained in the literature with reference to rods and beams [16], have been here numerically investigated with reference to a circular arch in presence of a single damage. Namely, it has been shown that, differently than what reported in the literature for rods and beams, in the case of arches the effects of single-damage severity and its location cannot be uncoupled. This unexpected result represents a further complication for the definition of damage identification procedures in arch structures.

As a further investigation a damaged dome arch structure, obtained as assemblies of circular arches, has been investigated highlighting how, in this kind of structures characterized by a polar symmetry, the variation of frequency and mode shapes can provide useful information for the presence of damage and its location.

2. The structural model and the equations of motion

Let an infinitesimal Timoshenko arch element, subtending an angle $d\phi$ at the centre of a circle of radius R, be considered as shown in Fig. 1. The cross-sectional properties are: area A, second moments of area I_1 and I_2, shear correction factor k', torsional constant J and polar moment of inertia I_p, all assumed to be constant. Furthermore the material properties, Young's modulus E, modulus of rigidity G and density per unit length ρ, are uniformly distributed. Although Fig. 1 shows the case of rectangular cross section, the following considerations hold for arbitrary cross section with a symmetry axis.

The degrees of freedom, in the space, with reference to both the in-plane and the out-of-plane motions, of the generic cross section are reported in Fig. 1. By considering the equilibrium of the arch element, in its undeformed configuration, and the linear elastic constitutive law, the differential equations of motion governing

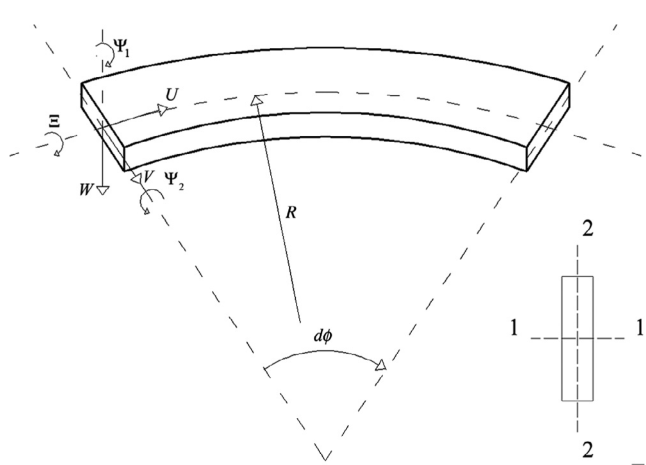

Fig. 1. The infinitesimal arch element and its cross section.

the in-plane free vibration of the Timoshenko arch have been derived by Chidamparam and Leissa [10] and by Howson and Jemah [3] which highlighted the procedure for the evaluation of the exact dynamic stiffness matrix. The exact vibration frequencies of the out-of plane behavior only, through an application of the Wittrick & William algorithm, have been obtained by Howson et al. [4]. The differential equations of motion governing both the in-plane and out-of-plane free vibrations of a circular Timoshenko arch have been re-written, in a unified approach by adopting a useful dimensionless form, by the present authors in a recent paper [26]. In that paper spatial undamaged frame dome structures have been analysed through an application of the Wittrick & Williams algorithm and a full parametric study of the in-plane and out-of plane circular Timoshenko arch has been presented as a function of independent dimensionless parameters. In this paper after recalling the main essential results reported in Ref. [26], useful for identifying the main arch parameters, Timoshenko spatial arch structures in presence of single or multiple concentrated damage are investigated.

With reference to the kinematic parameters shown in Fig. 1, the equations governing the in-plane motion of the arch are:

$$D(D^2+1+\gamma^2)U-(D^2+1-\gamma^2)V=0 \tag{1}$$

$$\begin{aligned}&\{\bar{\upsilon}D^4-(1+\lambda^2-2\bar{\upsilon}\gamma^2)D^2-\gamma^2(\lambda^2+1-\bar{\upsilon}\gamma^2)\}U+\\&-\{D[(1+\bar{\upsilon})D^2-\lambda^2+\gamma^2(1+\bar{\upsilon})]\}V=0\end{aligned} \tag{2}$$

where the symbol D^n denotes the n-th derivation with respect to polar coordinate ϕ, and the following dimensionless parameters have been introduced:

$$\lambda^2=\frac{AR^2}{I_1}\quad \gamma^2=\frac{\rho\omega^2R^2}{E}\quad \bar{\upsilon}=\frac{E}{k'G} \tag{3}$$

The differential equations governing the dynamic out of plane equilibrium of the considered Timoshenko arch turn out to be:

$$\begin{aligned}&(D^4+[(1+\bar{\upsilon})\gamma^2-\mu]D^2-(\lambda_2^2+\bar{\upsilon}\mu-\bar{\upsilon}\gamma^2)\lambda_2^2\gamma^2)W+\\&+(-D^2(1+\mu))\Xi=0\end{aligned} \tag{4}$$

$$(D^2(1+\mu)+\bar{\upsilon}\gamma^2(1+\mu))W-(-\mu D^2+1-\eta\gamma^2)\Xi=0 \tag{5}$$

where

$$\mu=\frac{GJ}{EI_2}\quad \eta=\frac{I_p}{I_2}\quad \lambda_2^2=\frac{AR^2}{I_2} \tag{6}$$

2.1. The evaluation of the natural frequencies and modes of vibration

The knowledge of the solution of the equations of motion described in [26] allows the evaluation of the exact dynamic stiffness matrix $\mathbf{K}(\omega)$ of the arch which relates the vector $\mathbf{P}$ of the nodal static parameters at the end of the element, to the corresponding vector of kinematic parameters $\mathbf{d}$ through the equation:

$$\mathbf{P}=\mathbf{K}(\omega)\mathbf{d} \tag{7}$$

Once the dynamic stiffness matrix of the structure is assembled, the natural frequencies can be calculated by means of a very effective method based on the Wittrick & Williams Algorithm [25].

The algorithm allows to evaluate the number of frequencies of vibration which are lower than a trial value ω_n and, therefore, by means of an iteration procedure, to converge to any required frequency.

This number turns out to be the sum of two terms which respectively represent the number of negative eigenvalues of the dynamic stiffness matrix evaluated at the specified frequency