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Sensitivity analysis of structural response to position of external applied load in plate flexural condition

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ABSTRACT

Procedures for discrete sensitivity analyses of the static responses of a structure, i.e., nodal displacements, mean compliance, local bending moments and stresses, are respectively developed for a plate or shell structure with regard to the position of an external load mainly because of the uncertainties related to the application point of a load in the preliminary design of an engineering structure. With the sensitivities developed herein, those responses can be estimated quickly and accurately by using the Taylor expansions after a position disturbance of the external load.

Based on the classical theory of the finite element analysis, the shift influences of a point or distributed load are firstly transformed into the associated value variation of the equivalent nodal forces. As a result of this transformation, the first- and second-order derivatives of the external load to its movement are constituted readily in a regularized formulation. Subsequently, the relevant sensitivities of the structural responses aforementioned are developed upon the essential concepts of the discrete approach. Finally, two typical examples are provided to illustrate the implementation of the sensitivity analyses derived and their applications in a quick estimate of the structural response variation. The numerical results show a high accuracy of the response sensitivity calculation.

1. Introduction

In the preliminary design stage of an engineering structure, there often exist some degrees of uncertainties associated not only with the structural parameters, e.g., the geometry dimensions, material characteristics, boundary conditions, but also with a part or all of the forces externally imposed. That is, the deterministic information of an applied load cannot be well provided with the value at a spatial location. Frequently, an external load may experience alterations about its magnitude and/or position during the design process so as to prove the load-bearing capacity of the resulting structure in various circumstances. Over the past decades, the computational mechanics analysis and also the design optimization of a structure with design dependent or independent loadings were implemented extensively in a variety of aspects [1–4]. Most commonly, the application point of an external force is fixed in the structural analysis process. However, the position of an external load may also be modified by designers/analysts, to some extent, to check against the various limitations of the stress (strength), displacement, compliance (stiffness), etc. In addition, it has also been noticed in practice that a small move or shift of the external load may bring about a substantial change on the structural performance and, therefore, the final optimal design of the structure. This phenomenon reveals that the situation of the external load movement is worthy of a more detailed investigation in the structural design. Nevertheless, there seems to be less information available on the structural behaviors with regard to the location variation of an external load. Then it is highly desirable for developing an effective procedure capable of estimating or assessing the structural performances quantitatively due to a small spatial move of the applied load. And the related sensitivity analysis presented in this paper can afford a quick and reliable solution to this problem.

On the other hand, when a structure is optimized by means of the topology, shape or sizing schemes, the external load locations are commonly assumed to be invariant in the optimization process. At the optimum, which is significantly affected by the initial distribution of the external loads, the resultant solution comes often in a critical or marginal situation of the design, i.e., the structural displacements at some special points reach the preset limitations; stresses in some regions arrive at the allowed levels or the mean compliance comes to the minimum, which corresponding to the maximum stiffness of the structure [5–8]. At times, it may become possible for the designer/analyst to examine the robustness and

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reliability of the final structural design by changing the external loads in various ways. In other words, the external applied loads may also experience alteration in magnitudes, positions or even directions, like the centrifugal forces [5,6,9–11], in order for them to assess the overall effect of these changes upon the design objective and constraint functions at the optimum. In this case, it is always most helpful to form a simple expression to deal efficiently with those responses. And it is well known that the associated derivatives or sensitivities can furnish the designer/analyst with the extremely useful information about the variations of those properties [12]. In fact, the sensitivity analyses of the structural responses with regard to the external load position are of fundamental importance to attack such types of problems in an efficient way. Moreover, they can also be utilized in an inverse problem of the load identification from the measured responses of structure whenever the direct measurement of a load is difficult or even impossible [13]. However, to the author's knowledge, the associated work still remains a challenge to the community about the analysis algorithms as well as the influences arising from the moves of the external applied loads.

This work is explored to conduct the discrete sensitivity analysis of the structural static responses to the motion of an external applied load in the plate/shell flexural situation by a further extension of the previous study in the 2D plane stress condition [14]. That is, the structure model chosen for the present treatment is assumed to primarily experience the out-of-plane displacements for flexural deformation, in addition to the in-plane deformation. Herein, it should be addressed that the sensitivity formulations cast previously by the author [14] are only a part of the present implementations of the related sensitivity analyses since the in-plane components are also included in the current developments. In this paper, the sensitivity formulations of the nodal displacement, structural compliance, local bending moment and stresses in an element specified are performed in sequence with respect to the move of an external applied load, concentrated or distributed. Whereas, the load's magnitude and direction (orientation), though they may be relevant to the applied position, are posed as constant in the location variation process. In the similar way to the previous study [14], the sensitivity analysis is still carried out by using the discrete method of the finite element (FE) analysis mainly due to the fact that these structural responses are usually obtained on the same discrete methodology. To this end, the structure is first discretized into an appropriate FE mesh. Then, based on the fundamental ideas of the FE analysis, the equivalent nodal forces of the external load are constructed by the adequate interpolation or shape functions of the plate-type element, in which the external load is applied. Then, due to this essential transformation, the influence of an external load shift is represented thoroughly by the associated magnitude variation of the equivalent nodal forces. Next, the explicit formulations of the first- and second-order derivatives of the equivalent forces to the continuous move of the application point of the external load are derived fairly with the aid of the properties of the plate element. Later, with the above derivations, the essential formulations of the sensitivity derivatives of the nodal displacements to the external load location are firstly developed using the direct method for the two types of loadings aforementioned. Furthermore, with the resulting load and displacement sensitivities, the sensitivity analyses for the compliance of the structure can be accomplished immediately. So are in sequence the sensitivities of the local bending moments and the stresses (normal and shear components) as well as the stress resultant, such as the principal stresses, within a finite element. As the structural analysis resorts most commonly to the numerical execution with the finite element method (FEM), such a derivation of the design sensitivity has an advantage of compatibility with the numerical assessments of the responses upon the same FE model. More importantly, it can be implemented easily in conjunction with existing commercial FE analysis packages, e.g., ANSYS.

In this paper, the four-node rectangular element is selected with five degrees of freedom (DOFs) per node. The Kirchhoff plate theory is used along with the plane stress model of bilinear interpolations for constructions of the derivatives of the equivalent nodal forces as well as the local stresses. In practical applications the rectangular plate or shell FE is most widely employed in structural optimization [2,4]. It will turn out that the derivatives of the external loads can be gained readily with the essential features of the usual shape functions of this plate element. Afterward, the computational procedures are further discussed to carry out the sensitivity analyses of both the structural global responses and even local ones in the interior of an element, respectively. Finally, the discrete sensitivity calculations of the structural responses will be illustrated and the result accuracies be demonstrated comprehensively with two typical examples. It can be seen that with the help of the sensitivities developed, the structural response variations due to moves of the external applied loads can be directly evaluated by use of the Taylor series expansions. The numerical solutions show that an excellent estimation of the responses can be achieved.

2. Sensitivity analysis of nodal displacements of a structure

As is well known, the thin plates or shells are widely used as structural components in many practices, such as the civil, automobile and aerospace engineering, etc. In this section, the discrete sensitivity analysis of the structural displacements to the position change of an external load is briefly discussed. Further details of the formulations will be presented in Section 4.

As for a general plate-type structure, modeled with an adequate FE mesh, the overall equations of the force equilibrium in terms of the nodal displacements are expressed as follows:

$$[K]\{u\} = \{P\} + \{F(s)\} \tag{1}$$

where $[K]$ is the system stiffness matrix. Without loss of generality, assume that all the kinematic boundary conditions have been accounted for in the assembly of the stiffness matrix such that $[K]$ is nonsingular. $\{u\}$ is a vector of the unknown structural nodal displacements, which is highly dependent on both the position and magnitude of the external loads. On the right hand of the equation, the external loads applied on the structure are decomposed into two parts on their characters: $\{P\}$ is the load invariable during the design process, whereas $\{F(s)\}$ is the nodal forces which is a function of the application position variable s. Apparently, a small position change of an external load will inevitably induce the redistribution of the internal forces in the structure, and thereby modify the structural responses, e.g., the nodal displacements, bending moments and so on. Taking partial derivatives of Eq. (1) with respect to a spatial location variable s of the external load can yield the following governing equation for the displacement derivatives:

$$[K]\frac{\partial\{u\}}{\partial s} = \frac{\partial\{F(s)\}}{\partial s} - \frac{\partial[K]}{\partial s}\{u\} \tag{2}$$

Suppose that the plate structure maintains itself in the case of a load moving, that is, the structure design is basically independent of movements of the external applied loads. Thus, the second term on the right-hand side of Eq. (2) vanishes. In consequence, the major computational effort is now focused on the first right differential term, i.e., the derivative of an applied load vector with respect to its position variation. After achievement of the relevant result, to be attained below, it becomes then a trivial task to get the first-order derivative sensitivity of the nodal displacements from Eq. (2)