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A comparative study of three composite implicit schemes on structural dynamic and wave propagation analysis

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ABSTRACT

A comparative study of three composite implicit schemes including the Bathe scheme (Bathe and Baig, 2005), the TTBDF scheme (Chandra et al., 2015) and the Wen scheme (Wen et al., 2017) is conducted in this paper. The stability and accuracy characteristics of these schemes are studied and compared by analytical and numerical simulation analysis. In the solution of wave propagation problems, a demonstrative dispersion analysis is given and problems are solved to illustrate the capabilities of the schemes for the solution of wave propagation problems as well as the numerical dissipation characteristics of the schemes. The nonlinear dynamic behavior of the Wen scheme is studied by considering commonly used nonlinear dynamic problems where the nonlinear performance of the scheme is exclusively compared with the Bathe and TTBDF schemes. The priority ranks of three presented composite schemes for different types of dynamic problems are obtained by theoretical and numerical analysis.

1. Introduction

In the last several decades, much research effort has been devoted to develop step-by-step time integration schemes for transient dynamic problems. Generally, there are two categories of schemes: implicit scheme and explicit scheme [1–4]. The implicit scheme is a good option for a practical use because it can give a stable solution with large time step, and the accuracy of numerical solutions can meet the usual requirements in engineering. A large amount of implicit schemes have been presented, see articles [5–7] and the references therein, and the representative implicit schemes such as the Newmark scheme [8], the HHT-α scheme [9], the generalized-α scheme [10] and the GSSS scheme [11] are widely used in actual engineering applications.

For step-by-step implicit schemes, numerical dissipation is important for dynamic analysis where numerical dissipation can filter out or to reduce the spurious, non-physical oscillations of high frequency modes induced by spatial discretization. In particular, for strong nonlinear problems, high-frequency numerical dissipation often improves the convergence of iterative equation solvers.

However, most of dissipative schemes have been developed in the context of linear elastodynamics [8–14], for the nonlinear analysis, it is observed that these schemes often fail to provide reliable high-frequency dissipation in the nonlinear analysis [15–17]. Therefore, in recent years, some composite schemes are developed to obtain more reliable numerical dissipation for linear and nonlinear problems [18–25], the intrinsic idea of these schemes is the use of sub-steps within one time step where both the non-dissipative and dissipative schemes are combined to obtain desirable calculation accuracy in low-frequency modes and numerical dissipation in high-frequency modes. For instance, the well-known composite time integration scheme proposed by Bathe and collaborators [18] is a simple combination of the non-dissipative trapezoidal rule and dissipative three-point Euler scheme. Although, this scheme, referred to as the Bathe scheme, requires roughly twice the computational cost as the trapezoidal rule per time step, shows better performance than the trapezoidal rule in nonlinear problems [26,27]. By use of the sub-step strategy similar to the Bathe scheme, a generalized robust composite time integration scheme is proposed by Dong [24] for nonlinear elastodynamics to overcome the loss of unconditional stability of the current time integration schemes in the nonlinear regime. To improve the numerical dissipation characteristics, the sub-step strategy from Bathe [26,27] and Dong [24] were extended to develop a new composite scheme which consists of three sub-steps where the trapezoidal

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rule is used to perform the first and second sub-steps, while the backward different formula is adopted to perform the third sub-steps [21]. For the rest of this paper, this new sub-step scheme will be referred to as the TTBDF scheme. Inspired by the work of Bathe and Baig [18], Wen et al. [22] have proposed a three sub-step scheme where the trapezoidal rule and the Euler backward formula is adopted for first and second sub-steps, respectively, but, for the third sub-step, the formula for the Houbolt scheme is adopted. This scheme, referred to as the Wen scheme, shows desirable characteristics in calculation accuracy and numerical dissipation.

A significant application of direct integration schemes is the wave propagation problem. In the solution of transient wave propagations, the errors stem from the spatial and temporal discretization which appear together and affect each other [28–32]. For temporal discretization, numerical dissipation is often used to filter out spurious oscillations, especially for high wave numbers. The introduced numerical dissipation should be large enough to suppress the high frequency spurious waves, and meanwhile, retain good accuracy for the low frequency waves. Bathe and coworkers have studied the dispersion properties of the Bathe scheme in the solution of wave propagation problems under different spatial discretization frameworks [32,33]. The Bathe scheme illustrates better performance than the trapezoidal rule in the wave propagation problems. In this paper, following the original work by Wen et al. [22], the dispersion properties of the Wen scheme are first investigated by considering the representative nonlinear dynamic problem and benchmark wave propagation problems.

The main objective of this paper is to study the nonlinear dynamics and wave propagation performance of the composite Wen and TTBDF schemes as well as the basic characteristics for structural dynamics, meanwhile, a comparative study of three dissipative composite schemes including the Bathe scheme, TTBDF scheme and Wen scheme is conducted. In the following, we introduce the considered composite schemes in Section 2 where the basic formulations of the presented composite schemes are given as well as the corresponding suggested algorithmic parameters. In Section 3, basic properties including stability and accuracy are illustrated and discussed. In Section 4, the dispersion errors of the composite schemes in the solution of 1D and 2D elastic wave propagations are exclusively studied under the same finite element discretization framework. Subsequently, in Section 5, several simulations are considered to comprehensively compare the performance of three composite schemes in linear and nonlinear dynamic problems and wave propagation problems.

2. The presented implicit time integration schemes

In the following analysis, the governing equation of motion for structural dynamics can be represented by

$$\boldsymbol{M}\ddot{\boldsymbol{u}} + \boldsymbol{C}\dot{\boldsymbol{u}} + \boldsymbol{K}\boldsymbol{u} = \boldsymbol{f} \tag{1}$$

where $\boldsymbol{u}$, $\dot{\boldsymbol{u}}$ and $\ddot{\boldsymbol{u}}$ are the nodal physical variables vector and its first and second derivative vectors with respect to time variable t. $\boldsymbol{M}$, $\boldsymbol{C}$ and $\boldsymbol{K}$ are the mass, damping and stiffness matrices, respectively. For nonlinear cases, $\boldsymbol{C}$ and $\boldsymbol{K}$ can be, respectively, obtained by $\boldsymbol{C} = \frac{\partial \boldsymbol{f}_d}{\partial \dot{\boldsymbol{u}}}$ and $\boldsymbol{K} = \frac{\partial \boldsymbol{f}_s}{\partial \boldsymbol{u}}$, where $\boldsymbol{f}_d$ and $\boldsymbol{f}_s$ are the nodal damping force and the elastic force vectors corresponding to the element internal stresses, respectively.

The considered time domain $[a, b]$ is divided by uniform time increment Δt into n sub time intervals $[t_i, t_{i+1}]$, where $i = 0, 1, 2, \ldots, n$; $\Delta t = \frac{b-a}{n}$.

The initial conditions for Eq. (1) are

$$\boldsymbol{u}(t_0) = \boldsymbol{d}_0, \quad \dot{\boldsymbol{u}}(t_0) = \boldsymbol{v}_0 \tag{2}$$

Another condition satisfying Eq. (1) is obtained as

$$\ddot{\boldsymbol{u}}(t_0) = \boldsymbol{M}^{-1}(\boldsymbol{f} - \boldsymbol{K}\boldsymbol{d}_0 - \boldsymbol{C}\boldsymbol{v}_0) \tag{3}$$

For the presented composite time integration schemes, the equation of motion is satisfied at discrete time $t + r\Delta t$ and $t + \Delta t$ within time interval Δt, where $r \in (0, 1)$.

2.1. The Bathe scheme

In this composite scheme [26,27], the complete time step Δt is subdivided into two sub-steps. For the first sub-step the trapezoidal rule is used; for the second sub-step the 3-point Euler backward scheme is employed with the resulting equations.

The basic formulation for the first sub-step is expressed as

$$\dot{\boldsymbol{u}}_{t+r\Delta t} = \dot{\boldsymbol{u}}_t + \frac{1}{2} r\Delta t(\ddot{\boldsymbol{u}}_t + \ddot{\boldsymbol{u}}_{t+r\Delta t}) \tag{4}$$

$$\boldsymbol{u}_{t+r\Delta t} = \boldsymbol{u}_t + \frac{1}{2} r\Delta t(\dot{\boldsymbol{u}}_t + \dot{\boldsymbol{u}}_{t+r\Delta t}) \tag{5}$$

$$\boldsymbol{M}\ddot{\boldsymbol{u}}_{t+r\Delta t} + \boldsymbol{C}\dot{\boldsymbol{u}}_{t+r\Delta t} + \boldsymbol{K}\boldsymbol{u}_{t+r\Delta t} = \boldsymbol{f}_{t+r\Delta t} \tag{6}$$

where $\boldsymbol{u}_t$, $\dot{\boldsymbol{u}}_t$ and $\ddot{\boldsymbol{u}}_t$ are the approximations of exact solution $\boldsymbol{u}(t)$, $\dot{\boldsymbol{u}}(t)$ and $\ddot{\boldsymbol{u}}(t)$, respectively. $\boldsymbol{u}_{t+r\Delta t}$, $\dot{\boldsymbol{u}}_{t+r\Delta t}$ and $\ddot{\boldsymbol{u}}_{t+r\Delta t}$ are the approximations of exact solution $\boldsymbol{u}(t + r\Delta t)$, $\dot{\boldsymbol{u}}(t + r\Delta t)$ and $\ddot{\boldsymbol{u}}(t + r\Delta t)$. $\boldsymbol{f}_{t+r\Delta t}$ is directly given by $\boldsymbol{f}(t + r\Delta t)$.

The basic formulation for the second sub-step is as follows:

$$\dot{\boldsymbol{u}}_{t+\Delta t} = c_1\boldsymbol{u}_t + c_2\boldsymbol{u}_{t+r\Delta t} + c_3\boldsymbol{u}_{t+\Delta t} \tag{7}$$

$$\ddot{\boldsymbol{u}}_{t+\Delta t} = c_1\dot{\boldsymbol{u}}_t + c_2\dot{\boldsymbol{u}}_{t+r\Delta t} + c_3\dot{\boldsymbol{u}}_{t+\Delta t} \tag{8}$$

$$\boldsymbol{M}\ddot{\boldsymbol{u}}_{t+\Delta t} + \boldsymbol{C}\dot{\boldsymbol{u}}_{t+\Delta t} + \boldsymbol{K}\boldsymbol{u}_{t+\Delta t} = \boldsymbol{f}_{t+\Delta t} \tag{9}$$

where $c_1 = \frac{1-r}{r\Delta t}$, $c_2 = \frac{-1}{(1-r)r\Delta t}$, $c_3 = \frac{2-r}{(1-r)\Delta t}$. $\boldsymbol{u}_{t+\Delta t}$, $\dot{\boldsymbol{u}}_{t+\Delta t}$ and $\ddot{\boldsymbol{u}}_{t+\Delta t}$ are the approximations of exact solution $\boldsymbol{u}(t + \Delta t)$, $\dot{\boldsymbol{u}}(t + \Delta t)$ and $\ddot{\boldsymbol{u}}(t + \Delta t)$. This composite scheme is desirable for nonlinear problems, and attractive because only the usual symmetric stiffness, mass and damping matrices are used, and no additional unknown variables (i.e., Lagrange multipliers) need to be solved for. This composite scheme is available in the ADINA program [18]. Here the splitting parameter $r = 1/2$ is selected with due consideration of computation efficiency and coding implementation [18].

2.2. The TTBDF scheme

The TTBDF scheme is formed by using the trapezoidal rule for the first and second sub steps and a BDF-like algorithm in the third sub step [21], the time step for each sub step is $\delta t = \Delta t/3$. The basic formulations for the first and second sub steps, respectively, are

$$\boldsymbol{u}_{t+\frac{\Delta t}{3}} = \boldsymbol{u}_t + \frac{\Delta t}{6}\left(\dot{\boldsymbol{u}}_t + \dot{\boldsymbol{u}}_{t+\frac{\Delta t}{3}}\right) \tag{10}$$

$$\dot{\boldsymbol{u}}_{t+\frac{\Delta t}{3}} = \dot{\boldsymbol{u}}_t + \frac{\Delta t}{6}\left(\ddot{\boldsymbol{u}}_t + \ddot{\boldsymbol{u}}_{t+\frac{\Delta t}{3}}\right) \tag{11}$$

$$\boldsymbol{M}\ddot{\boldsymbol{u}}_{t+\frac{\Delta t}{3}} + \boldsymbol{C}\dot{\boldsymbol{u}}_{t+\frac{\Delta t}{3}} + \boldsymbol{K}\boldsymbol{u}_{t+\frac{\Delta t}{3}} = \boldsymbol{f}_{t+\frac{\Delta t}{3}} \tag{12}$$

and

$$\boldsymbol{u}_{t+\frac{2\Delta t}{3}} = \boldsymbol{u}_{t+\frac{\Delta t}{3}} + \frac{\Delta t}{6}\left(\dot{\boldsymbol{u}}_{t+\frac{\Delta t}{3}} + \dot{\boldsymbol{u}}_{t+\frac{2\Delta t}{3}}\right) \tag{13}$$

$$\dot{\boldsymbol{u}}_{t+\frac{2\Delta t}{3}} = \dot{\boldsymbol{u}}_{t+\frac{\Delta t}{3}} + \frac{\Delta t}{6}\left(\ddot{\boldsymbol{u}}_{t+\frac{\Delta t}{3}} + \ddot{\boldsymbol{u}}_{t+\frac{2\Delta t}{3}}\right) \tag{14}$$