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Size-dependant behaviour of functionally graded microplates based on the modified strain gradient elasticity theory and isogeometric analysis

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ABSTRACT

This paper presents a robust numerical model, which takes into account both size-dependent and shear deformation effects, for the bending, buckling and free vibration analyses of functionally graded microplates. The size-dependent effect is captured by using the modified strain gradient elasticity theory with three length scale parameters, whilst the shear deformation effect is accounted by using the third-order shear deformation theory. The rule of mixture is employed to describe the distributions of material phrases through the plate thickness. By using Hamilton's principle, the governing equations are derived and then discretized by employing an Isogeometric Analysis (IGA) approach, where the Non-Uniform Rational B-Splines (NURBS) basis functions are adopted to meet the C^2-continuity requirement. Physical mesh convergence and verification studies are performed to prove the accuracy and reliability of the present model. In addition, parametric studies are also carried out to investigate the size effect in conjunction with the influences of gradient index, shear deformation effect and boundary conditions on the responses of microplates.

1. Introduction

In recent years, Functionally Graded Materials (FGMs) known as advanced materials have been broadly investigated in the field of structural mechanics. Being made from a mixture of different phrases of materials with their properties varying continuously through the thickness, Functionally Graded (FG) structures do not have stress concentrations found in laminated composite counterparts. The pioneering work on FGMs was carried out by a group of Japanese material scientists [1,2]. Typically, a FG structure consists of two material phrases, which are technically called ceramic and metal. The ceramic component is low thermal conductivity and thus has higher temperature-resistant, whereas the metal one is more ductile to prevent thermal stress causing fractures. FGMs have not only used in macro-scale [3–12] but also in nano- and micro-scale applications such as thin films [13,14], atomic force microscopes (AFMs) [15], micro- and nano-electro-mechanical systems (MEMS and NEMS) [16]. It is worth noting that microbeams and microplates are the fundamental structures broadly employed in AFM, MEMS and NEMS. Having the small-size features, the behaviour of such structures are considerably affected by size-dependant phenomena, which were verified experimentally.

The size effects were observed in plastic deformation of some metals in the work of Guo et al. [17] and Poole et al. [18]. A considerable size-dependency was observed in the work of Chong and Lam [19] for epoxy and Lam et al. [20] for epoxy polymeric beams. Additionally, a remarkable discrepancy between the experimental and numerical results of microbeams obtained from classical beam theory was reported in the work of McFarland and Colton [21]. Liu et al. [22] carried out a micro-torsion test and they found out that the reduction of wires diameter results in an increase in the torsion strength of thin copper wires. Overall, the aforementioned experimental studies revealed that the classical elasticity theory fails to predict accurately the behaviour of structures at micro-scale. This could be attributed to the presence of material length scale parameters which used to account for size-dependant phenomena. Consequently, there have been a number of theories developed to account for the size effects, in which the length scale parameters are involved in the constitutive equations. One of the first high-order elasticity theories is classical couple stress theory proposed by Toupin [23], Mindlin and Tiersten [24] and Koiter [25]. It has

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two material length scale parameters for isotropic elastic materials. The Modified Coupled Stress theory (MCT) proposed by Yang et al. [26] considered a high-order equilibrium equation, which results in a symmetric couple stress tensor. Consequently, only one material length scale parameter associated with rotation gradient involved in the constitutive equations. Another class of high-order elasticity theory were introduced by Mindlin [27], in which the first- and second-order gradients of strains tensor are included in the strain energy expression. Mindlin and Eshel [28] suggested a modified theory which is only first-order gradient of strain tensor along with five additional material length scale parameters applied for isotropic linear elastic materials. Altan and Aifantis [29] developed a simplified version of the high-order elasticity theory, which involves only one material length scale parameter. Based on the theory of Mindlin [27], Fleck and Hutchinson [30–32] introduced a new theory called the strain gradient theory. The Modified Strain gradient elasticity Theory (MST) was proposed by Lam et al. [20] by modifying the classical strain gradient theory of Mindlin [27] and Mindlin and Eshel [28] to establish a new set of high-order metrics, where the number of additional length scale parameters was reduced from five to three. Moreover, the MST can be reduced to MCT if two of the three material length scale parameters regarding to dilatation gradient and deviatoric stretch gradient are taken to be zero. In order to analyse the structural behaviour micro-structures, literature reveals that the MCT and MST are widely used. Comparing to the former, the latter theory is more general since it covers dilatation and deviatoric stretch gradient tensors in addition to rotation gradient tensor and classical strain tensor as in the MCT.

The MST has been employed to analyse the size-dependent behaviour of microbeams and microplates. Kong et al. [33] and Wang et al. [34] studied the static and dynamic behaviour of small scale beams in accordance with Euler-Bernoulli beam and Timoshenko beam theories. Akgöz and Civalek [35] investigated the buckling response of microbeams. The strain gradient Euler-Bernoulli and Timoshenko beam theories were also developed by Kahrobaiyan et al. [36] and Zhang et al. [37], respectively. Li et al. [38] proposed a size-dependant bilayered Bernoulli-Euler beam model and addressed the locations of neutral and zero-stress axes. Linear and nonlinear vibrations of FG Timoshenko microbeams were carried out by Ansari et al. [39]. Based on Timoshenko beam theory, Shenas et al. [40] conducted vibration analysis of FG microbeam carrying microparticles in the thermal environment. The static bending, instability and free vibration analyses of simply supported microplates based on Kirchhoff assumptions were studied by Wang et al. [41]. They pointed out that the size effects are dismissed if the plate thickness is greater than the material length scale parameter about 15 times. Ashoori Movassagh and Mahmoodi [42] employed the extended Kantorovich method to derive approximate closed form solutions for bending behaviour of rectangular Kirchhoff microplates with simply supported and clamped boundary conditions. The results of their work revealed that the material length scale parameter associated with the dilatation gradient has the most influence on bending response of the microplates, while the least effect is found for the parameter corresponded to stretch deviatoric gradient. Sahmani and Ansari [43] used high-order shear deformation plate theory to predict the response of FG microplates with simply supported boundary conditions. Analytical solutions for the bending problems of bi-layered Kirchhoff microplate was derived by Li et al. [44]. Ansari et al. [45] also developed a Mindlin plate model to investigate the behaviour of FG circular and annular microplates based on the MST. By using the refined plate theory, Zhang et al. [46,47] presented studies on the behaviour of rectangular FG microplates resting on elastic foundation and circular/annular FG microplates. The thermoelastic damping of FG microplates was also investigated by Emami and Alibeigloo [48]. Study on the free vibration responses of FG quadrilateral microplates in thermal environment based on Mindlin plate model was conducted by Shenas and Malekzadeh [49]. Overall, those investigations pointed out a considerable size-dependant behaviour of microbeams and microplates, especially when thickness of the structures is on the same order of the material length scale parameter. In addition, it is seen that most of the aforementioned studies were carried out based on analytical approaches, which were limited to certain types of boundary conditions, loading patterns and geometries. Recently, Mirsalehi et al. [50] employed a finite strip method to study the buckling and free vibration of FG thin square microplates. However, their work is only applicable for thin FG microplates due to the adoption of Kirchhoff theory.

The IGA approach was firstly introduced by Hughes et al. [51] in 2005. Since then, it quickly becomes a hit in many fields of computational mechanics, where its efficiency compared to traditional Finite Element Analysis (FEA) was proven [52]. The fundamental concept of the IGA is to bridge the gap between the methods for analysis and conventional computer-aided design tools using NURBS basis functions. Therefore, the time taken from preliminary designs to analysis progress is reduced considerably while exact geometries of the modelled objects are preserved. The compelling advantages of the IGA have been proved through a large number of publications for plate problems [53–75]. Having distinguished features, the NURBS basis functions are capable of providing a smooth and high continuity interpolation, which allows to construct the elements in a straightforward manner. The shear locking phenomenon can also be reduced with higher order-degree basis functions [62]. Furthermore, circular and annular plates as well as plates with complicated cutouts were successfully modelled with exact geometry by using IGA techniques [57,52,61,74,5,43,54,73,55–59,64,65,71,63,75].

Although the IGA approach has been successfully applied to investigate the size-dependant behaviour of FG plates based on the non-local elasticity theory [76] and MCT [77,78], no effort has been devoted to extend this efficient approach to FG microplates based on the MST. The main objective of this study is therefore to develop a robust numerical model used to investigate the bending, free vibration and buckling responses of FG microplates in accordance with the MCT. The third-order shear deformation theory of Reddy [79] is adopted to takes into account the shear deformation effect, while the effect of material variations through the plate thickness is considered using the rule of mixture. Hamilton's principle is used to construct the weak form equations. Then, the NURBS basis functions are employed to interpolate the displacement field and the geometries of rectangular and circular microplates. Physical mesh convergence and verification studied are also carried out to show the reliability and the accuracy. Furthermore, parametric studies are also carried out to investigate the size effect in conjunction with the influences of gradient index, shear deformation effect and boundary conditions on the responses of microplates.

2. Theoretical formulation

2.1. Modified strain gradient elasticity theory

Lam et al. [20] modified the strain gradient theory by decomposing the second-order deformation gradients into three parts (the dilatation gradient vector ζ_i, the deviatoric stretch gradient tensor $\eta^{(1)}_{ijk}$ and the rotation gradient tensor χ^s_{ij}) using three different internal length scales. The second-order deformation metrics are mutually independent and only the dilatation scalar and the dilatation gradient vector depend on volumetric deformation. Dilatation gradient is 1st-order tensor, which implies the dilatations in each direction. The deviatoric stretch gradient is 3rd-