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Dynamics framework for 2D anisotropic continuum-discrete damage model for progressive localized failure of massive structures

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ABSTRACT

We propose a dynamics framework for representing progressive localized failure in materials under quasi-static loads. The proposed model exhibits no mesh dependency, since localization phenomena are taken into account by using the embedded strong discontinuities approach. Robust numerical tool for simulation of discontinuities, in which the displacement field is enhanced to capture the discontinuity, is combined with continuum damage representation of FPZ-fracture process zone. Based upon this approach, a two-dimensional finite element model was developed, capable of describing both the diffuse damage mechanism accompanied by initial strain hardening and subsequent softening response of the structure. The results of several numerical simulations, performed on classical mechanical tests under slowly increasing loads such as Brazilian test or three-point bending test were analyzed. The proposed dynamics framework is shown to increase computational robustness. It was found that the final direction of macro-cracks is predicted quite well and that influence of inertia effects on the obtained solutions is fairly modest especially in comparison among different meshes.

1. Introduction

One of the most important reasons that can cause structural failure is material micro-cracking evolving into localized collapse mechanisms (see [12,25]). The simulation of the behavior of structures and components with discontinuities has become the topic of much interest for the current research in the field of computational mechanics. Several theories have been provided the fundamental foundation for dealing with the simulation of the onset and propagation of cracks in material, both at macroscopic and microscopic levels. Generally speaking, the presently available approaches to model discontinuities can be classified into two main families: the fracture mechanics approach and the continuum mechanics approach. However, it is well documented in [19,7] that using classical continuum mechanics models for post-localization studies where strain-softening phenomena appear is unreliable. Consequently, to overcome the shortcomings of local theories for modeling strain-softening, in the context of continuum mechanics-based models, the embedded discontinuity approach (EDA) was recently introduced giving rise to two variants of weak embedded discontinuity formulations and strong embedded discontinuity formulations. In the former case, with representative works in [22,28], the strain field becomes discontinuous, but the displacement field remaining continuous, across the limits of a narrow band (strain localization band). Alternative approach concerns the case when the strain localization band collapses into a surface, so-called displacement discontinuity. The displacement field that becomes discontinuous across that surface implies that the strain field becomes unbounded (e.g. [1,2,6,9,11,16,20,24,27,30]). Yet another alternative method is the extended finite element method (XFEM), in which a global approximation to the strong discontinuity kinematics is supplied by exploiting the partition of unity property of the shape functions (see [8]). In comparison to XFEM, the embedded discontinuity method has more computational advantage. Namely, in the approximation of the displacement field, XFEM requires additional nodal degrees of freedom, while in EDA the additional degree of freedom can be eliminated by static condensation at the element level, so that the dimension of the discretized problem does not increase at global level. As a consequence, for the efficiency reasons, the embedded strong discontinuity method is chosen in this work.

The vast majority of the previous studies using the embedded discontinuity approach only considered quasi-static problems.

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Fairly few works in dynamics were carried out with this approach, such as [13] or [5]. As the main novelty here, we present a two-dimensional model with the main contributions as follows:

- Capability of representing the localized failure of massive structure in dynamics by taking into account combination of strain hardening in FPZ-fracture process zone and softening with embedded strong discontinuities.
- Providing an alternative X-FEM approach to modeling failure phenomena in dynamics with a more robust implementation, and a more reliable prediction of final crack direction for massive structures with a significant contribution of FPZ.
- A multi-surface damage model including normal interface and tangential interface damage modes, as generalization of mode I and mode II failure modes in LFM-Linear Fracture Mechanics.

The paper is organized as follows: Section 2 is devoted to the theoretical formulation of the combined continuum damage-embedded strong discontinuity model, followed by Section 3 in which the numerical implementation is discussed. In Section 4, we present the results of numerical simulations performed on classical mechanical tests such as Brazilian test or three-point bending test, and analyze. Finally, Section 5 closes the paper with some concluding remarks.

2. Theoretical formulation

2.1. Continuum damage model

The damage model with isotropic hardening presented herein is based on the idea first proposed in [21] where the internal variable is chosen as the fourth order compliance tensor, $\bar{\mathbf{D}}$ representing an anisotropic damaged state. The damage criterion prescribing the admissible values of stress in the sense of damage is defined:

$$\bar{\Phi}(\boldsymbol{\sigma},\bar{q}) = \hat{\bar{\Phi}}(\boldsymbol{\sigma}) - (\bar{\sigma}_f - \bar{q}) \leqslant 0 \tag{1}$$

where $\bar{\sigma}_f$ refers to the limit of elasticity indicating the first cracking and $\bar{q}$ is a stress-like hardening variable which handles the damage threshold evolution.

The internal energy of such a damage model can formally be written:

$$\overline{\chi}(\bar{\boldsymbol{\varepsilon}},\bar{\mathbf{D}},\bar{\xi}) = \frac{1}{2}\bar{\boldsymbol{\varepsilon}}\bullet\bar{\mathbf{D}}^{-1}\bar{\boldsymbol{\varepsilon}} + \bar{\Xi}(\bar{\xi}) \tag{2}$$

where $\bar{\xi}$ is the hardening variable, and $\bar{\Xi}(\bar{\xi})$ stands for the corresponding hardening potential.

By using the Legendre transformation to exchange the roles between the stress and deformation, we further introduce the complementary energy potential:

$$\overline{\psi}(\boldsymbol{\sigma},\bar{\mathbf{D}},\bar{\xi}) = \boldsymbol{\sigma}\bullet\bar{\boldsymbol{\varepsilon}} - \overline{\psi}(\bar{\boldsymbol{\varepsilon}},\bar{\mathbf{D}},\bar{\xi}) = \frac{1}{2}\boldsymbol{\sigma}\bullet\bar{\mathbf{D}}\boldsymbol{\sigma} - \bar{\Xi}(\bar{\xi}) \tag{3}$$

By exploiting the second law of thermodynamics, we can obtain the explicit form of the damage model dissipation defined as follows:

$$0 \leqslant \bar{\mathcal{D}} = \boldsymbol{\sigma}\bullet\dot{\bar{\boldsymbol{\varepsilon}}} - \dot{\bar{\psi}}(\bar{\boldsymbol{\varepsilon}},\bar{\mathbf{D}},\bar{\xi}) = \dot{\boldsymbol{\sigma}}[-\bar{\boldsymbol{\varepsilon}} + \bar{\mathbf{D}}\boldsymbol{\sigma}] + \frac{1}{2}\boldsymbol{\sigma}\bullet\dot{\bar{\mathbf{D}}}\boldsymbol{\sigma} - \frac{d\bar{\Xi}(\bar{\xi})}{d\bar{\xi}}\dot{\bar{\xi}} \tag{4}$$

In the case of "elastic" process where $\dot{\bar{\mathbf{D}}} = 0$ and $\dot{\bar{\xi}} = 0$, the dissipation inequality (the Clausius-Duhem inequality) above becomes an equality, $\bar{\mathcal{D}} = 0$, and leads to the appropriate form of constitutive equations for damage model can be established:

$$\bar{\boldsymbol{\varepsilon}} = \bar{\mathbf{D}}\boldsymbol{\sigma} \Rightarrow \boldsymbol{\sigma} = \bar{\mathbf{D}}^{-1}\bar{\boldsymbol{\varepsilon}} = \frac{\partial\overline{\psi}(\bar{\boldsymbol{\varepsilon}},\bar{\mathbf{D}},\bar{\xi})}{\partial\bar{\boldsymbol{\varepsilon}}};;\bar{q} = -\frac{d\bar{\Xi}(\bar{\xi})}{d\bar{\xi}} \tag{5}$$

By assuming that those results remain valid for an inelastic process in which the internal variables are now modified, $\dot{\bar{\mathbf{D}}}\neq 0$ $\dot{\bar{\xi}}\neq 0$, we can define the positive damage dissipation:

$$0 < \bar{\mathcal{D}} = \frac{1}{2}\boldsymbol{\sigma}\bullet\dot{\bar{\mathbf{D}}}\boldsymbol{\sigma} + \bar{q}\dot{\bar{\xi}} \tag{6}$$

In order to obtain the associated evolution equations for internal variables, the principle of maximum dissipation has to be enforced under the constraints $\bar{\Phi}(\boldsymbol{\sigma},\bar{q}) \leqslant 0$. By introducing Lagrange multiplier $\bar{\gamma}$, the previous problem can then be recast as the unconstrained maximization problem:

$$\max_{\bar{\Phi}(\boldsymbol{\sigma},\bar{q})\leqslant 0}[\bar{\mathcal{D}}(\boldsymbol{\sigma},\bar{q})] \Longleftrightarrow \max_{\dot{\bar{\gamma}}\geqslant 0}\min_{\forall(\boldsymbol{\sigma},\bar{q})}\underbrace{[-\bar{\mathcal{D}}(\boldsymbol{\sigma},\bar{\mathbf{D}},\bar{\xi}) + \dot{\bar{\gamma}}\bar{\Phi}(\boldsymbol{\sigma},\bar{q})]}_{\bar{\mathcal{L}}(\boldsymbol{\sigma},\bar{\mathbf{D}},\bar{\xi},\bar{q})} \tag{7}$$

Applying the Kuhn-Tucker optimality conditions for the chosen damage Lagrangian the evolution equations for the internal variables can thus be written:

$$\begin{aligned} 0 &= \frac{\partial\bar{\mathcal{L}}(\boldsymbol{\sigma},\bar{\mathbf{D}},\bar{\xi},\bar{q})}{\partial\boldsymbol{\sigma}} \Rightarrow \dot{\bar{\mathbf{D}}} = \dot{\bar{\gamma}}\frac{\partial\bar{\Phi}(\boldsymbol{\sigma},\bar{q})}{\partial\boldsymbol{\sigma}}\otimes\frac{1}{\boldsymbol{\sigma}} \\ 0 &= \frac{\partial\bar{\mathcal{L}}(\boldsymbol{\sigma},\bar{\mathbf{D}},\bar{\xi},\bar{q})}{\partial\bar{q}} \Rightarrow \dot{\bar{\xi}} = \dot{\bar{\gamma}}\frac{\partial\bar{\Phi}(\boldsymbol{\sigma},\bar{q})}{\partial\bar{q}} \end{aligned} \tag{8}$$

Since $\hat{\bar{\Phi}}(\boldsymbol{\sigma})$ is a homogeneous function of degree one, which implies that $\frac{\partial\bar{\Phi}}{\partial\boldsymbol{\sigma}}\boldsymbol{\sigma} = \hat{\bar{\Phi}}(\boldsymbol{\sigma})$, from (8) we can write the evolution of damage model compliance as:

$$\dot{\bar{\mathbf{D}}} = \dot{\bar{\gamma}}\frac{\partial\bar{\Phi}}{\partial\boldsymbol{\sigma}}\otimes\frac{\partial\bar{\Phi}}{\partial\boldsymbol{\sigma}}\frac{1}{\hat{\bar{\Phi}}(\boldsymbol{\sigma})} \tag{9}$$

The corresponding loading-unloading criteria are also a part the Kuhn-Tucker optimality conditions:

$$\dot{\bar{\gamma}} \geqslant 0, \bar{\Phi}(\boldsymbol{\sigma},\bar{q}) \leqslant 0, \dot{\bar{\gamma}}\bar{\Phi}(\boldsymbol{\sigma},\bar{q}) = 0 \tag{10}$$

Finally, the consistency condition which takes form as $\dot{\bar{\gamma}}\dot{\bar{\Phi}}(\boldsymbol{\sigma},\bar{q}) = 0$, will lead to the corresponding value of damage multiplier.

$$\dot{\bar{\gamma}} = \frac{\frac{\partial\bar{\Phi}}{\partial\boldsymbol{\sigma}}\bullet\bar{\mathbf{D}}^{-1}\dot{\bar{\boldsymbol{\varepsilon}}}}{\frac{\partial\bar{\Phi}}{\partial\boldsymbol{\sigma}}\bullet\bar{\mathbf{D}}^{-1}\frac{\partial\bar{\Phi}}{\partial\boldsymbol{\sigma}} + \bar{K}\left(\frac{\partial\bar{\Phi}}{\partial\bar{q}}\right)^2} \tag{11}$$

where $\bar{K}$ is the hardening modulus for the bulk material.

With these results in hand, we can easily write the stress rate constitutive equations for the continuum damage model (see Fig. 1):

$$\dot{\boldsymbol{\sigma}} = \begin{cases} \bar{\mathbf{D}}^{-1}\dot{\bar{\boldsymbol{\varepsilon}}} & \dot{\bar{\gamma}} = 0 \\ \left[\bar{\mathbf{D}}^{-1} - \frac{(\bar{\mathbf{D}}^{-1}\frac{\partial\bar{\Phi}}{\partial\boldsymbol{\sigma}})\otimes(\bar{\mathbf{D}}^{-1}\frac{\partial\bar{\Phi}}{\partial\boldsymbol{\sigma}})}{\frac{\partial\bar{\Phi}}{\partial\boldsymbol{\sigma}}\bullet\bar{\mathbf{D}}^{-1}\frac{\partial\bar{\Phi}}{\partial\boldsymbol{\sigma}} + \bar{K}\left(\frac{\partial\bar{\Phi}}{\partial\bar{q}}\right)^2}\right]\dot{\bar{\boldsymbol{\varepsilon}}} & \dot{\bar{\gamma}} > 0 \end{cases} \tag{12}$$

2.2. Discrete damage model

The damage model of this kind is further enhanced to be able to describe localized failure leading to softening. The localized failure is represented by a strong discontinuity in the displacement field across the surface Γ_s (see Fig. 2). Therefore, the total displacement field can be written as the sum of a continuous regular part $\bar{\mathbf{u}}(\mathbf{x},t)$ and a discontinuous irregular part corresponding to the displacement jump $\bar{\bar{\mathbf{u}}}(\mathbf{x},t)$ (see also [31] and [32]) (see Fig. 3):

$$\mathbf{u}(\mathbf{x},t) = \bar{\mathbf{u}}(\mathbf{x},t) + \bar{\bar{\mathbf{u}}}(\mathbf{x},t)H_{\Gamma_s}(\mathbf{x}) \tag{13}$$

where $H_{\Gamma_s}(\mathbf{x})$ denotes the Heaviside function (see Fig. 4):

$$H_{\Gamma_s}(\mathbf{x}) = \begin{cases} 1 & \mathbf{x}\in\partial\Omega^- \\ 0 & \mathbf{x}\in\partial\Omega^+ \end{cases} \tag{14}$$