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An energy-momentum co-rotational formulation for nonlinear dynamics of planar beams

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ABSTRACT

This article presents an energy-momentum integration scheme for the nonlinear dynamic analysis of planar Euler-Bernoulli beams. The co-rotational approach is adopted to describe the kinematics of the beam and Hermitian functions are used to interpolate the local transverse displacements. In this paper, the same kinematic description is used to derive both the elastic and the inertia terms. The classical midpoint rule is used to integrate the dynamic equations. The central idea, to ensure energy and momenta conservation, is to apply the classical midpoint rule to both the kinematic and the strain quantities. This idea, developed by one of the authors in previous work, is applied here in the context of the co-rotational formulation to the first time. By doing so, we circumvent the nonlinear geometric equations relating the displacement to the strain which is the origin of many numerical difficulties. It is rigorously shown that the proposed method conserves the total energy of the system and, in absence of external loads, the linear and angular momenta remain constant. The accuracy and stability of the proposed algorithm, especially in long term dynamics with a very large number of time steps, is assessed through four numerical examples.

1. Introduction

Dynamics of slender beams is still a very active research field especially when it comes to large deformations and displacements. Flexible beams are used in many applications, for instance large deployable space structures, aircrafts and wind turbines propellers, offshore platforms. These structures undergo large displacements and rotations and in some cases moderate-to-large strains. Several approaches are available to model the dynamics of geometrically flexible nonlinear beams. In addition to the Total Lagrangian approach [1–5], floating approach [6–8] and co-rotational ones [9–26] have been considered for the development of efficient formulations. Whilst the total Lagrangian approach can be considered as the natural setting for geometrically exact dynamics, the co-rotational method is still an attractive approach to derive highly nonlinear beam elements because it combines accuracy with numerical efficiency. Especially for very large structures with a high number of beam elements, efficiency is still of great importance for successful simulation.

Response of large structures to earthquake, to impact or to extreme loading conditions are some examples where dynamics is essential with efficiency being a key ingredient that decides about the choice of the finite element. Here, long term stability is a fundamental feature of a time integration method to capture extended responses over sufficiently long time intervals. Implicit time stepping methods are often used together with nonlinear finite elements to investigate complex dynamic problems. It is well known that Newmark's method [37] and alike are conditionally stable for nonlinear dynamics. To avoid these instabilities, Geradin and Cardona [38] introduced numerical dissipations (Alpha method [39]) in order to damp the high frequencies with the consequence that the system energy is not conserved [40,41]. Since the early work of Simo and Tarnow [41], it is accepted that energy conservation, respectively control, is key for stability. In their work, they presented a methodology to construct time integration algorithms that inherit, by design, the conservation of momenta and energy for geometrically nonlinear problem involving quadratic Green-Lagrange strains. Generally, the design of energy-momentum conserving algorithms comes with conservation of linear and angular momentum as well, hence the term energy-momentum methods. The core idea of these methods is to use a discrete directional derivative to construct scheme that preserve the Hamiltonian along with other integrals. This concept can be

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traced back to Gotusso [42] and was first applied to elastodynamics by Gonzalez [43]. Since then much effort was devoted to develop energy-momentum methods for various types of formulations and structural elements. For nonlinear rod dynamics we refer to [48–52] and contributions to nonlinear shell dynamics have been made in [53–57], among others. In all cases, some form of shearable structures were considered, that is, either the Reissner-Mindlin kinematic for shells or the equivalent Timoshenko one for the rod. Nonlinear dynamics of hypoelastic continuum has been addressed by [43–45]. Further energy-momentum-related work is that of Betsch and Steinmann [46]. A simple parameter free collocation-type composite time integration scheme has been proposed by Bathe [47] with the objective to conserve energy. Of special interest, also with regard to this work, is the formulation by Sansour et al. [54,55], which is designed to secure energy conservation independently of the nonlinear complexities involved in the strain-displacement relations. It has been applied to arbitrary continuum formulations [61] and to geometrically exact Bernoulli beam model [49]. Gams et al. [50] developed a time integration algorithm in the spirit of the method described in [61] for the geometrically exact planar Reissner beam. Besides, they considered the dissipation of high frequency oscillations associated to energy-momentum methods [62].

With regard to the co-rotational formulation for rods, we employ here the one originally proposed by Rankin and Nour-Omid [58,59], and further developed by Battini and Pacoste [30,31] and many other authors. The fundamental idea of a co-rotational formulation is to decompose the large motion of the element into rigid body and pure deformation parts through the use of a local system which continuously rotates and translates with the element. The deformation is captured at the level of the local reference frame, whereas the geometric nonlinearity induced by the large rigid-body motion, is incorporated in the transformation matrices relating local and global quantities. The main interest is that the pure deformation part can be assumed as small and can be represented by a linear or a low order nonlinear theory. Avoiding the nonlinear relationship between the strain tensor and the displacement gradient is what makes the co-rotational approach very attractive and efficient for nonlinear static analysis. For a general account, we refer also to [27–36].

As one may expect, there have been many efforts to develop energy-momentum methods for co-rotational formulations as well. These efforts have been only partially successful. Examples of previous attempts are that of Crisfield and Shi [9] who developed a mid-point energy-conserving time integrator for co-rotating planar trusses. In their formulation, the time-integration strategy is closely linked to the co-rotational procedure which is "external" to the element. A similar approach was applied to the dynamic of co-rotational shell [24] and laminated composite shells [25]. Yang and Xia [26] proposed the energy-decaying and momentum-conserving algorithm in the context of thin-shell structures. Galvanetto and Crisfield [11] applied the previously developed energy-conserving time-integration procedure to implicit nonlinear dynamic analysis of planar beam structures. Various end- and mid-point time integration schemes for the nonlinear dynamic analysis of 3D co-rotational beams are discussed in [18]. They concluded that the proposed mid-point scheme is an "approximately energy conserving algorithm". Le et al. [12] adopted Interpolation Interdependent Element formulation [60], hence cubic interpolation functions, to derive both the inertia and elastic terms in conjunction with a Newmark-type time integration algorithm and considering simplifications in the expression of the mass matrix. Le et al. [12] showed that this formulation is more efficient than using constant mass matrices as it requires less elements. The formulation was extended to 3D Bernoulli beam elements without [19,20] and with warping [21]. Salomon et al. [22] showed the conservation of energy and momentum in the 2D analysis. But, they did not get exact angular momentum conservation in the 3D analysis.

It was soon recognized that the decomposition of the beam motion into a rigid and deformation-related parts with the help of a local frame that moves with the beam produces complex kinetic energy terms as a result of the movement of the local frame regardless of order of the interpolating functions. To circumvent these difficulties, Iura et al. [13] proposed to use an inertial frame to derive kinetic energy function in terms the global displacement components. Similar approach has been followed by Crisfield et al. [10,18] who suggested to derive the mass matrix by interpolating global quantities with linear shape functions (Timoshenko model). The use an inertial frame to derive kinetic energy function in terms the global displacement components was also recommended in Crisfield et al. [18] as a remedy to complicated expressions of kinetic energy-related terms.

In all the above examples energy conservation is either approximately achieved or enforced by means of constraint equations. Indeed, so far no method exists which inherently fulfills the conservation properties of energy and momenta in the context of co-rotational formulation. It is with this goal in mind that we approach the present research. At the heart of the approach is to apply the fundamental ideas of Sansour et al. [54,55] in the context of the present co-rotational formulation. The complexities induced by the decomposition of the beam movement have hampered the development of a consistent energy-momentum conserving co-rotational formulation. While the fundamental idea of Sansour et al. [54,55] can be summarized as using the strain-displacement relations to deduce strain rate quantities with the help of which then a strain filed is integrated using the same schemes as for the displacement fields, the task as such is not as straightforward as it may seem. The choice of the correct strain rates is crucial since multiple nonlinear relations exist between the displacements and further quantities which constitute the strain field. Questions arise as to which of the nonlinear quantities are to be integrated first. Also and beyond the possible formulation, the applicability of the same in long-term dynamics is to be tested as well.

The outline of the paper is as follows. In Section 2, the kinematics and strain measures of the 2D beam element are shortly presented. Section 3 is devoted to the Hamilton's principle and the conserving properties. In Section 4, the energy momentum scheme is developed, the element (i.e. the elastic and inertia terms) is fully derived. Proofs of the conservation of energy, linear and angular momenta are given in Section 5. In Section 6, four numerical applications are presented in order to assess the performances of the proposed method. The paper concludes in Section 7.

2. Beam kinematics and strain definition

2.1. Co-rotational beam kinematics

The kinematics of the beam and all the notations used in this section are shown in Fig. 1. The motion of the element is decomposed in two parts. In a first step, a rigid body motion is defined by the global translation (u_1, w_1) of the node 1 as well as the rigid rotation α. This rigid motion defines a local coordinate system (x_l, z_l) which continuously translates and rotates with the element. In a second step, the element deformation is defined in the local coordinate system. Assuming that the length of the element is properly selected, the deformational part of the motion is always small relative to the local co-ordinate systems. Consequently, the local deformations can be expressed in a simplified manner. The vectors of global and local displacements are now defined by