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Overlapping finite elements for a new paradigm of solution

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ABSTRACT

We present novel overlapping finite elements for a new paradigm of solution proposed in our previous papers, see Bathe (2016) and Bathe and Zhang (2017).

We give the formulation of the new overlapping elements and the solutions of basic numerical examples to investigate the robustness and efficiency of the new finite elements. The results show that the new overlapping elements are quite distortion insensitive and the numerical integration of the element matrices is efficient. The computational effort to integrate the matrices is much less than in meshfree methods. Finally, we illustrate the complete solution scheme of the new paradigm using the overlapping finite elements in the analysis of the bracket problem already considered in Bathe and Zhang (2017).

While the paper proposes and studies new overlapping elements, we conclude that further research is needed to fully harvest the potential of the new analysis approach.

1. Introduction

The finite element method is now established as an effective numerical procedure and is much used for the analysis of structures, fluids, and multi-physics problems, see for example [3]. However, despite its great success, the required meshing still presents difficulties. In engineering analyses of complex components, oftentimes, much more time is spent on reaching an adequate mesh than obtaining the solution of the finite element model. In a traditional finite element analysis, the elements must abut each other and cannot overlap, which leads to meshing difficulties and frequently to highly distorted elements that pollute the accuracy of the overall finite element solution.

For these reasons, many meshfree or meshless methods have been developed, see for example Refs. [2,4]. In a meshfree method, the global solution field is constructed using scattered points in the analysis domain without a mesh. Compared with analyses using traditional finite elements, the solutions obtained using a meshfree method may be relatively insensitive to the points or nodes used. Hence the solution accuracy may be improved. However, while the effort to establish the discretization is much less, and good solution accuracy can be obtained, the required numerical integration using a meshfree method is computationally expensive [2,4–9]. This expense largely restricts the wide use of meshless methods in engineering practice.

To improve the accuracy of solutions obtained when using traditional finite elements, specifically when distorted meshes are employed, some researchers have focused on improving the performance of the traditional finite elements by generalizing finite element formulations, see for example [10–13]. In a related approach, interpolation covers were introduced in Refs. [14,15] to obtain improved solution accuracy. Compared with the use of meshfree methods, these schemes are quite efficient in the numerical integration. However, the procedures need a mesh and the solutions are still quite sensitive to mesh distortions, indeed the solution accuracy may decay rapidly when highly distorted elements are used.

To significantly improve the meshing procedures of geometrically complex solids, we proposed a new paradigm for CAD driven simulations [1,2]. In the new approach, the CAD part is immersed in a Cartesian grid of uniform cells determined by the analyst, the boundary of the part is discretized, and the cells within the analysis domain of the CAD part are automatically, and with very little computational effort, converted to traditional finite elements. Thereafter, overlapping finite elements are used along the boundaries of the part to fill-in the empty space and couple with the traditional finite elements.

In the new paradigm of solution, the meshing, including the clean-up of the geometry, is embedded into CAD driven solutions. Undistorted finite elements are used in the inner part of the analysis domain and overlapping elements are used on and near the boundaries. Hence, provided the overlapping finite elements perform well, the scheme does not have an element distortion problem, and compared with meshfree methods, the computational

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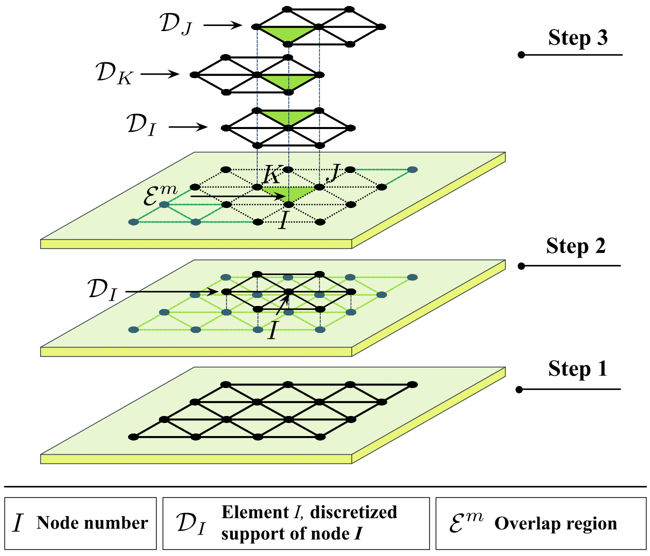

Fig. 1. Schematic of elements corresponding to $\mathcal{D}_I$, $\mathcal{D}_J$, $\mathcal{D}_K$ using triangular regions; the local fields are constructed for the support of each node I, J, K; the global field is built from the local fields.

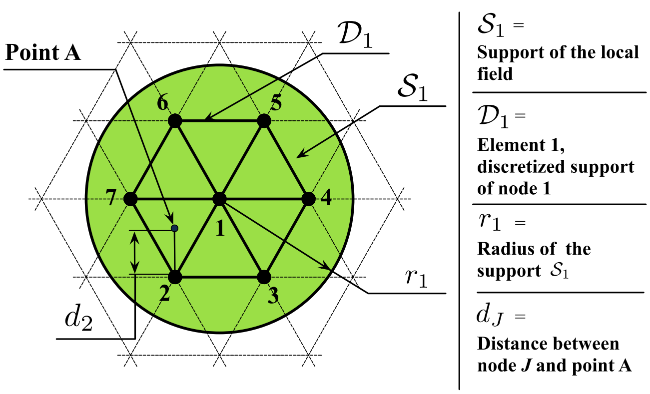

Fig. 2. Schematic of the 7-node element 1, or discretized support $\mathcal{D}_1$.

time is much less because overlapping elements are only used near the boundaries.

In Ref. [2] we used the spheres of the method of finite spheres [16,17] as overlapping elements, and also pointed out that other overlapping elements could be used. Clearly, to fully harvest the potential of the given analysis approach, efficient overlapping elements are needed.

Our objective in this paper is to present new robust and efficient overlapping finite elements that satisfy the consistency requirements and use much less integration points, and hence are, for example, much more efficient than those in Refs. [2,18]. In Section 2, we give the basic theory of the new overlapping elements including the theory for coupling the overlapping elements to the traditional finite elements. Then in Section 3, we discuss the solutions of four numerical examples in which we compare the performance of the new overlapping elements with the performance of the method of finite spheres [16,17] and the finite element method enriched by interpolation covers [14,15]. Thereafter, to illustrate the complete solution scheme of the new paradigm for analysis using the new overlapping elements, we revisit in Section 4 the analysis of the bracket already considered in Ref. [2]. Finally, we present our conclusions in Section 5.

2. The overlapping finite elements

Our objective in this section is to present the theoretical formulation of the new overlapping elements including the theory for coupling these with the traditional finite elements, the imposition of the Dirichlet boundary conditions and the numerical integration used for the element matrices.

2.1. The interpolations used for the overlapping finite elements

Effective overlapping elements should show two important properties. Firstly, the integration of the element stiffness matrices should be computationally efficient, and secondly, the overlapping elements should be distortion-insensitive. In order to develop overlapping elements that show both properties, we propose a new scheme for the local and global approximation fields. Considering Fig. 1, there are three major steps.

The first step is to discretize "the region" in the usual way. We use here 3-node triangular elements but other elements could also be used. In the new paradigm of solution, the region considered is the space between the boundary of the discretized CAD part and the Cartesian mesh of traditional finite elements [2]. The second step is to construct the local field within the support of each node. The local analysis domain $\mathcal{D}_I$ (in Fig. 1, given by the union of the 6 triangular regions) denotes the support of node I, and we refer to it as element I. In order for the scheme to be quite distortion insensitive, the local field is constructed as discussed in Section 2.1.1.

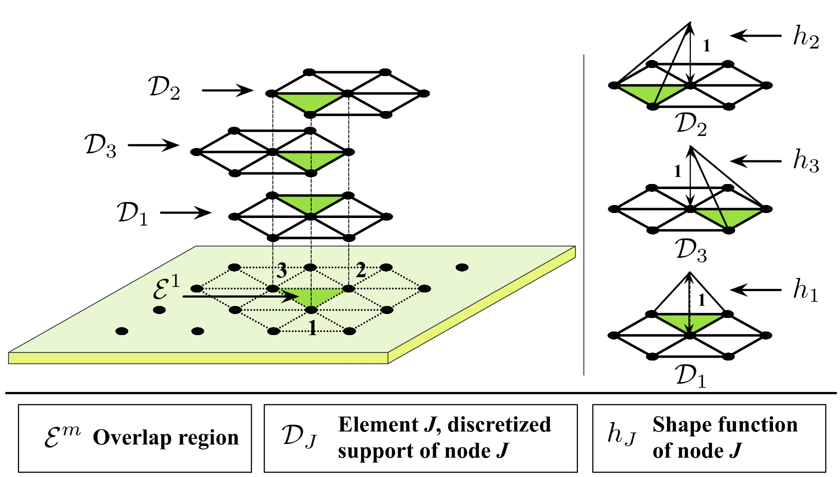

Fig. 3. Schematic of the interpolation process in the overlap region $\mathcal{E}^1$.