Computers and Structures 187 (2017) 88–100

Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

A regularized non-smooth contact dynamics approach for architectural masonry structures

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ARTICLE INFO

Article history:
Received 20 September 2016
Accepted 15 February 2017

Keywords:
Non-Smooth Contact Dynamic (NSCD)
Measure Differential Inclusion (MDI)
Associative friction
Masonry
Rigid blocks
Dynamic analysis

ABSTRACT

A Non-Smooth Contact Dynamic (NSCD) formulation is used to analyze complex assemblies of rigid blocks, representative of real masonry structures. A model of associative friction sliding is proposed, expressed through a Differential Variational Inequality (DVI) formulation, relying upon the theory of Measure Differential Inclusion (MDI). A regularization is used in order to select a unique solution and to avoid problems of indeterminacy in redundant contacts. This approach, complemented with an optimized collision detection algorithm for convex contacts, results to be reliable for dynamic analyses of masonry structures under static and dynamic loads. The approach is comprehensive, since we implement a custom NSCD simulator based on the Project Chrono C++ framework, and we design custom tools for pre- and post-processing through a user-friendly parametric design software. Representative examples confirm that the method can handle 3-D complex structures, as typically are architectural masonry constructions, under both static and dynamic loading.

1. Introduction

An advanced rigid-body dynamics formulation is used to analyze assemblies of rigid blocks, representative of architectural masonry constructions, under static and dynamic loadings. The method is interfaced with design custom tools for pre-processing and post-processing through a user-friendly parametric design software, which allows the design of complex masonry structures in the three-dimensional space.

While the proposed model assumes blocks to be very stiff, it focuses on the reliable description of associative friction laws at the contact surfaces. This approach was introduced for masonry constructions by Kooharian [27], who envisioned the possibility of studying structures of this kind within the plasticity theory. Under the assumption of unilateral constraints and absence of tensile strength, limit analysis was used to calculate the load which causes instability at the contact surfaces between the blocks [23]. The reliability and advantages of this approach are founded on the characteristics of this type of structures, which are prone to instability failure because of the definite prevalence of compressive strength over tensile strength. Meanwhile, other analyses that require the exact knowledge of the material parameters are difficult to be applied, because masonry is a composite material for which the nature of the composing blocks and interlayers, as well as their interactions, is highly irregular and, therefore, uncertain. Experiments [6,7] have provided evidence that, when a great number of blocks is organized into very complex arrangements, the stress percolation results to be highly localized, evidencing unloading islands in a stress stream.

Compared to the thrust-line graphical method [13,10], still used in the current practice for preliminary analyses, the dynamical formulations of the problem, as indicated by Livesley and Gilbert [30,21], provide significant advances because all the possible types of movements are considered and the interactions between all blocks can be fully appreciated. In particular, the method proposed here is set within the category of the Non-Smooth Contact Dynamic (NSCD) framework, firstly developed by Moreau [32] to handle specifically unilateral constraints. This provides a proper definition of contacts, whose value can be clarified comparing the NSCD approach with alternative mathematical formulations, namely the Ordinary Differential Equations (ODE) and the Differential Algebraic Equations (DAE) formulations [19], which have been more often applied to masonry. The DAE approach is the more refined and expresses constraint equations together with the differential equations, as it happens in the classical multi-

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http://dx.doi.org/10.1016/j.compstruc.2017.02.002
0045-7949/

body dynamics at the base of most Discrete Element Method software (DEM) [12]. In methods of this kind, contacts are modeled with penalty functions, which represent spring-damper elements whose flexibility can be adjusted to match the real stiffness of the contact surfaces, for instance using the Hertz-Mindlin theory or similar models. However, a physically accurate compliance of contact points with high stiffness coefficients results in steep penalty functions, something that would require extremely short time steps in most ODE or DAE integrator algorithm, at the point of being very inefficient or even unusable [22].

More specifically, modeling very rigid blocks provides the opportunity to reproduce the stick-slip transition, representing a sudden change in motion at collision. This is a typical contact phenomenon that strongly affects the failure mechanism and the corresponding ultimate load. Our model, by leveraging on the theory of Measure Differential Inclusions (MDI) [34,33], describes forces and accelerations as distributions of measures, while velocities are functions of Bounded Variation (BV), not necessarily continuous. Instead, the aforementioned alternative approaches describe velocities through smooth functions, which therefore cannot represent the sudden changes in motion at collision [1]. Despite workarounds have been proposed [35,20], they actually detriment the clarity of the model and the initial advantage of those methods in terms of computational effort.

The NSCD framework implemented here has been developed by one of the authors within the Project Chrono, a multi body dynamics C++ library [31]. As in the Fortran implementation of [25], the time integration method is stable even under large time steps, and the user has to set just the mass and friction parameters of the material. It should be remarked that modeling masonry blocks as perfectly rigid contacts may seem idealized, but it does not decrease the quality of the model. In fact, for the reasons mentioned earlier, stiffness and damping laws are affected by local complex geometric and rheological phenomena, that cannot be assessed, even limiting to average physical values, without an *ad hoc* experimental research on its own. Moreover, even the most classical solution for linear elastic bodies under concentrated contact forces suffers from intrinsic inconsistencies [17]. To our knowledge, only recently the NSCD formulation for rigid blocks has started to be successfully applied to the study of old, possibly deteriorated, masonry construction [29], but many variations are possible within this broad class of models.

Especially, the friction law used in the proposed approach deserves further comments. According to experimental results [50,9], friction is slightly associative because of roughness of the contact profiles, i.e., a normal displacement (dilatancy) accompanies sliding across the frictional surface [18]. However, it is clear since the work by Drucker [14] that sliding in the presence of friction *à la* Coulomb invalidates the general bounding theorems of plasticity, since the normality rule is not fulfilled. The formulation of the problem is complicated, and a right failure load may be associated with an incorrect failure mode [21]. Our approach includes set-valued force laws and complementarity constraints as required by the original Coulomb contact model. This is formulated as a Differential Variational Inequality (DVI). As such, DVIs impose constraints in the form of Variational Inequalities (VI) during the time evolution of the system [38,37]. Such set-valued functions can be expressed by the same MDI theory.

A common issue in NSCD methods as applied to masonry structures [15,28,41] is the multiplicity of solutions for the contact forces, especially in the tangential direction. This is a natural consequence of the rigid body idealization, although it often does not affect the uniqueness of solutions for speeds and trajectories. Here, we introduce a regularization that ensures uniqueness even for contact forces, resulting in better numerical performance of the time-stepping algorithm and in improved clarity of the plotted results.

When one deals with architectural complex masonry structures, not only the simulation time, but even the geometrical definition of blocks and the communicability of the results can be a problem. This is why we have integrated our computational software with a userfriendly design tool. We used the Grasshopper@ free parametric design plug-in for the Rhino@ CAD software, both to generate and modify the geometry of the source data and to post-process the computational results. With such a tool we provide the real-time visualization of forces, stress and collapse mechanisms, and displays the *effective* thrust line in arches, as the envelope curve of the resultant of the contact forces at the blocks interfaces.

The plan of the article is as follows. In Section 2 we present the proposed method and its numerical implementation, with special focus on the contact frictional model. In Section 3, the potentiality of the method is highlighted through the analysis of representative case studies. The efficiency of the computations is addressed in Section 4, where we study the response to dynamic loads. The overall achievements, drawbacks and further developments are discussed in the concluding Section 5.

2. Non-smooth contact dynamics

In a classical ODE or DAE, one assumes smooth speeds and accelerations. However, the introduction of hard contacts leads to non-smooth trajectories, and this requires a NSCD framework based on MDI, that encompasses jumps in speeds. In a MDI, acceleration is not a function in a classical sense because, as a consequence of impact events and other impulsive phenomena, it contains a certain number of spikes, which can be considered using the theory of (vector signed) measure distributions. In detail, positions $\boldsymbol{q}(t)$ are *Absolutely Continuous* (AC) functions but speeds $\boldsymbol{v}(t)$ are *functions of Bounded Variation* (BV), with finite variation $\bigvee_{t_a}^{t_b} \boldsymbol{v}(t)$ for $[t_a, t_b] \subset [0, T]$, i.e., they do not need to be absolutely continuous or even continuous.

Before proceeding with the mathematical model of our NSCD problem, we need to introduce some definitions.

Definition 1. A Variational Inequality VI($\boldsymbol{F}, \mathcal{K}$) is a problem of the type

$$\boldsymbol{x} \in \mathcal{K} : \quad \langle \boldsymbol{F}(\boldsymbol{x}), \boldsymbol{y} - \boldsymbol{x} \rangle \geqslant 0 \quad \forall \boldsymbol{y} \in \mathcal{K}, \tag{1}$$

with $\mathcal{K}$ closed and convex, and $\boldsymbol{F}(\boldsymbol{x}) : \mathcal{K} \to \mathbb{R}^n$ continuous.

Definition 2. The dual cone $\mathcal{K}^*$ of $\mathcal{K}$ is a convex cone expressed as:

$$\mathcal{K}^* = \{\boldsymbol{y} \in \mathbb{R}^n : \langle \boldsymbol{y}, \boldsymbol{x} \rangle \geqslant 0 \quad \forall \boldsymbol{x} \in \mathcal{K}\}. \tag{2}$$

Definition 3. A Cone Complementarity Problem CCP($A, \boldsymbol{b}, \Upsilon$) is the problem of finding a $\boldsymbol{x}$ that satisfies

$$A\boldsymbol{x} - \boldsymbol{b} \in \Upsilon^*, \quad \boldsymbol{x} \in \Upsilon, \quad \langle A\boldsymbol{x} - \boldsymbol{b}, \boldsymbol{x} \rangle = 0, \tag{3}$$

where Υ is a (convex) cone. One can also use the notation $A\boldsymbol{x} - \boldsymbol{b} \in \Upsilon^* \perp \boldsymbol{x} \in \Upsilon$. The CCP is equivalent to a VI where $\mathcal{K} = \Upsilon$ and with affine $\boldsymbol{F}$.

2.1. System state

For each i-th block in the system, we introduce the position $\boldsymbol{x}_i \in \mathbb{R}^3$ of its center of mass, and we introduce its rotation matrix $A_i \in \mathsf{SO3}$, both expressed relatively to the absolute reference. To avoid redundant parameters, we parametrize $\mathsf{SO3}$ using its double