Computers and Structures 187 (2017) 101–113

Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

Full extended layerwise method for the simulation of laminated composite plates and shells ☆

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A R T I C L E I N F O

Article history:
Received 31 July 2016
Accepted 25 October 2016
Available online 14 April 2017

Keywords:
Composites
Extended layerwise theory
Delamination
Transverse crack

A B S T R A C T

In the extended layerwise method (XLWM) established in the previous studies (Li et al., 2015; Li, 2016), the delamination front has to be consistent with the element edges. However, the general delamination damage region detected by the nondestructive evaluation (NDE) has extremely complex front, it should be very difficult to exactly model and analysis general delamination damage region in the XLWM. A full extended layerwise method (Full-XLWM) is developed in this study to avoid the dependence of the delamination region on the finite elements. In the Full-XLWM, the delamination is simulated by the Heaviside function and branch function of the displacement field in the thickness direction, whilst transverse cracks are modeled by the extended finite element method (XFEM). The weak discontinuous function along the thickness direction is adopted to consider the interlaminar interfaces. The delamination front is independent on the meshing as the level set function is employed, so the analysis model of delamination can be established from a uniform finite element meshing. In the numerical examples, the static analysis of the composite plates and spherical shells are investigated for the straight and circular delamination front.

1. Introduction

The transverse cracks and delaminations resulted from the impact loads are the most important problems in applications of composite structures. Choi et al. [3] reported that intraply matrix cracking is the initial damage mode. The delaminations initiates once the matrix cracks reach the interface between the ply groups having different fiber orientations, due to the local stress concentrations at the matrix crack tips. The transverse cracks and delaminations of laminated composites are the internal damages and not visual inspected, they would seriously affect the structure stiffness and strength. The suddenly failure will inevitably occur as the internal damages accumulate in the absence of immediate inspection and repair.

Over the past years, although many methods and models have been proposed for the prediction of transverse cracks [4–18] and delamination [19–22]. In the existing methods, it is assume that the transverse crack is complanate and perpendicular to middle surface of plates, but that was not case, especially the transverse crack in composite laminates. There are very little research works focus on the problem in where two kinds of damages coexist simultaneously [23,24]. O'Brien [25] derived an equation for the strain energy release rate (SERR) of the local delamination growth from a matrix ply crack. Dharani and Tang [26] developed a micromechanics analytical model for the fracture behaviour of the fibre reinforced composite laminate with a transverse matrix crack and longitudinal debonding along 0°/90° interface. Both the matrix and fibres are considered as linear elastic materials, and a consistent shear lag theory is used to represent the stress-displacement relations. Nairn and Hu [27] extended a variational mechanics method of micro-cracking damage in cross-ply laminates to account for the delaminations emanated from the micro-cracks tips. This new 2D stress analysis method is used to calculate the total strain energy, effective modulus, and longitudinal thermal expansion coefficient. Takeda and Ogihara [28] studied the delamination initiation and growth from the transverse crack tips in toughened-type CFRP cross-ply laminates by the replica technique. Berthelot and Corre [29] developed an analytical model to evaluate the stress distributions in cross-ply laminates subjected to tensile loading and containing the transverse cracks and delamination. Swindeman [30] studied the coupled modeling of matrix cracks and delamination in laminated composite materials based on the finite element

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☆ Supported by the Natural Science Foundations of China (11502286) and the Fundamental Research Funds for the Central Universities (3122017021, 3122017034).

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http://dx.doi.org/10.1016/j.compstruc.2016.10.023
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method and experimentally validated. Damage initiation is determined using the LARC03 failure criterion [31,32]. Delamination along ply interfaces is modeled using cohesive zones model (CZM).

As the extended finite element method (XFEM) was presented to solve the problems that exhibit strong and weak discontinuities in material and geometric behavior, it was recently applied to simulate delamination problems in composites [33–39]. Recently, XFEM was applied to simulate delamination growth coupled with the VCCT [40,41] or CZM [42–45]. Although the shell elements method improved by XFEM were applied to model thick-through cracks or delaminations individually for the composite laminated structures, there is very little work has yet been reported for the typical damage pattern including matrix crack and delamination [24]. The typical damage pattern of composite laminated plates is a complex 3D crack with layered characteristics. Since it is very difficult to apply XFEM directly to deal with complex 3D crack, we can convert this complex 3D crack with layered characteristics to two 2D crack by using an appropriate displacements model along thickness direction.

A XLWM was developed for the laminated composite beams, plates and shells with delamination and transverse crack by the layerwise method and XFEM in our previous studies [1,2]. The displacement field of the XLWM in the thickness direction is constructed with the linear Lagrange interpolation functions, the one-dimensional (1D) weak discontinuous function and strong discontinuous function. The strong and weak discontinuous functions are applied in the displacement field along the thickness direction to model the displacement discontinuity induced by the delaminations and strain discontinuity induced by the interface between the layers, respectively. The transverse cracks are simulated in the in-plane displacement discretization. The XLWM of laminated composites can not only perfectly describe the multiple delaminations together with the thick through or non-thick through transverse cracks, but also accurately obtain the displacement and stress fields of the transverse crack tips and delamination front. The weak discontinuous function is used to ensure the C_0 continuous condition at the interlaminar interfaces. Because the XLWM is quasi-3D and the transverse cracks of each single layer are independently described, the distribution of the stress intensity factor (SIF) along the thickness direction can be calculated, and the predicted crack growth angle is different for each mathematic layer. This serves an important advantage compared with the existing shell elements enriched by the XFEM. The XLWM expanded the application of the XFEM in the damage analysis and prediction of composite laminated structures. In addition, the modeling capabilities of the XLWM are essentially the same as the conventional 3D displacement finite element method. Therefore, the existing fracture mechanics method based on the conventional 3D displacement finite element method can be conveniently applied to XLWM.

Although our previous works about XLWM had solving some restrictive obstacles of applying XFEM to the typical damage pattern of composite laminated plates, there are two weaknesses need to be conquered in the next investigations. One of the weaknesses is that the XFEM employed in XLWM to simulate the transverse crack is a traditional methods for the orthotropic materials, for example, the optimal convergence rates does not be guaranteed. The other weakness is that only the Heaviside function is applied to the delamination in the existing XLWM, it means that the delamination is simulated by the node pairs. The delamination front has to consistent with the element edges, so the delamination region is depended on the finite elements and the front is approximated by the short straight lines (element edges). Furthermore, the general delamination damage region usually detected by the nondestructive evaluation (NDE) in the engineering applications, the extremely complex shape of the crack front should result into the modeling and analysis difficult in the XLWM.

The goal of the proposed investigation is to overcome the second aforementioned drawback of existing XLWM and reduce the modeling difficult. The branch function is introduced into the displacement field in thickness direction to develop a full extended layerwise method (Full-XLWM) for the composite plates and shells. In the proposed Full-XLWM, the branch function is used to simulate the delamination front though the elements and its singularity of stress fields, while the Heaviside function is applied to the delaminated interfaces. The level set is used to trace the delamination front in the Full-XLWM, so the delamination front is independent on the meshing and the analysis model of delamination can be obtained from a uniform finite element meshing.

2. Full-XLWM of the composite laminates

2.1. Displacement fields of Full-XLWM

For the composite laminated plates and shells with multiple delaminations, the displacements field proposed in the present study is schematically shown in Fig. 1, where h_k is the thickness of the k-th layer and z_k is the coordinate of the interface between k-th layer and $(k-1)$-th layer in thickness direction. In Fig. 1, the numbers on the left side denote the nodes of the displacements field along the thickness direction, while the numbers on the right side denote the interfaces between the layers. The displacements at point $(\xi,\vartheta,\varsigma)$ in the composite laminates with multiple delaminations can be expressed as

$$u_\alpha(\xi,\vartheta,\varsigma,t)=\sum_{k=1}^{N+2}\phi_k(\varsigma)u_{\alpha ik}(\xi,\vartheta,t)+\sum_{k=1}^{N_D}\Xi_k(\varsigma)u_{\alpha lk}(\xi,\vartheta,t)+\sum_{k=1}^{N}\Theta_k(\varsigma)u_{\alpha rk}(\xi,\vartheta,t)+\sum_{k=1}^{N}\Upsilon_k(\varsigma)u_{\alpha tk}(\xi,\vartheta,t) \tag{1}$$

where $\alpha=1,2,3$ denotes the components in the ξ,ϑ and ς directions. $u_{\alpha ik}$, $u_{\alpha lk}$, $u_{\alpha rk}$ and $u_{\alpha tk}$ are the nodal freedoms, the additional nodal freedoms to model displacements discontinuity induced by delaminations, the additional nodal freedoms to model strains discontinuity induced by interface between the layers and the additional nodal freedoms with respect to the crack tip, respectively. The subscripts i, l, r and t denote the standard nodal freedom, the additional nodal freedom for delaminations, the additional nodal freedom for interfaces and the additional nodal freedom for crack tip, respectively. N is the number of the mathematical layers of the composite laminated plate. It can be seen from Fig. 1 that the numbers of the standard freedoms and the additional freedoms for interfaces are $N+2$ and N, respectively. N_D is the number of nodes which have to be enriched to model the delaminations. Let $\Phi_{ik}=\phi_k(\varsigma)$, and ϕ_k is the linear Lagrange interpolation functions along the thickness direction of the composite laminated plate. Let $\Phi_{rk}=\Theta_k(\varsigma)$, and $\Theta_k=\phi_k(\varsigma)\chi_k(\varsigma)$ is the weak discontinuity shape function used to model the strains discontinuity in the interface between the layers, where $\chi_k(\varsigma)$ is the one-dimensional signed distance function. Let $\Phi_{lk}=\Xi_k(\varsigma)$, and $\Xi_k=\phi_k(\varsigma)H_k(\varsigma)$ is the shape function used to model delaminations, where $H_k(\varsigma)$ is the one-dimensional Heaviside function. Let $\Phi_{tk}=\Upsilon_k(\varsigma)$, and Υ_k is given by

$$\Upsilon_k=\phi_k(\varsigma)\sqrt{r}\sin\frac{\theta}{2} \tag{2}$$

where $r=\sqrt{h^2+\varsigma^2}$, $\theta=\arctan(\varsigma/h)$, h is a signed distance function, see Fig. 2 [35].

Since Υ_k is discontinuous on the crack line ($\theta=\pm\pi$), see Fig. 3, it is the branch function and can describe the discontinuity resulted from the crack line. This branch function cannot exactly reconstruct the displacement field near the crack tip (delamination