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An effective adaptive time domain formulation to analyse acoustic–elastodynamic coupled models

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ABSTRACT

This work is concerned with the development of an effective technique to model the propagation of interacting acoustic and elastic waves. Here, spatial discretization by finite elements is adopted, and uncoupled analyses of acoustic and elastodynamic subdomains are considered, with the interaction between the different subdomains of the model being accomplished by interface forces. Adaptive explicit and implicit time-marching techniques are employed, in which the time integrators are locally computed, assuming different values along the spatial and temporal discretizations, as the solution evolves. The proposed solution algorithm is very efficient, accurate, flexible and easy to implement, standing as a very attractive approach. At the end of the paper, numerical results are presented, illustrating the good performance and potentialities of the new methodology.

1. Introduction

Time domain numerical modelling of wave propagation in highly heterogeneous media requires robust simulation algorithms in order to preserve accuracy, efficiency and stability. In some configurations, this situation becomes even more complex, and different types of waves interact through different subdomains, requiring interface routines to be introduced to properly analyze the coupled model. This is the case considering acoustic-elastodynamic interacting domains. In this context, several numerical difficulties may arise, and very poor results may be obtained if proper techniques are not considered.

In the context of the Finite Difference Method, for instance, Lombard and Piraux [1] list the following main reasons for low confidence results in a situation where there is discontinuity of physical properties: (i) spurious diffractions occur due to the stair-step representation of arbitrarily shaped interfaces [2]; (ii) reduction of the convergence order due to the non-smoothness of the solution across the interfaces, leading to numerical instabilities even for low contrast physical parameters [3]; (iii) the jump conditions and the boundary conditions are not incorporated in the schemes, so that the conversion, refraction and diffraction wave phenomena are not correctly described [4]; etc. In fact, several problems may arise taking into account the simulation of coupled acoustic-elastodynamic models according to the numerical techniques that are employed. In addition to these difficulties, efficiency is another issue that must be dealt with properly. In this case, accuracy and stability may restrict the time-step size to very small values, which are adequate to subdomains with high wave propagation velocities, severely damaging the efficiency of the analysis. Moreover, the coupled systems of equations that arise, taking into account all subdomains of the model, may get too extensive, demanding a lot of computational resources and becoming very ineffective (or even prohibitive), also severely damaging the efficiency of the methodology. In fact, developing proper numerical techniques to analyze wave propagation through coupled acoustic and elastodynamic media is a challenging field, and several efforts have been applied on the topic, allowing a vast literature to be available nowadays [5–28].

This work proposes a new formulation to analyze coupled acoustic-elastodynamic models. As one will observe, the proposed new procedure is quite effective, eliminating most of the thorny issues related to this type of problem, as well as providing very accurate and efficient analysis. Here, the spatial discretization of the interacting acoustic and elastodynamic subdomains is carried out taking into account the Finite Element Method (FEM) [29,30]. The time domain analysis of the hyperbolic systems of equations that arise, once spatial discretization by standard finite elements is considered, is carried out taking into account the new adaptive methodology proposed by Soares [31]. In this methodology, two

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time integration parameters are considered, namely α and γ, which are allowed to assume different values at each FEM element and at each time step. The computation of the γ parameter is designed to improve the accuracy and to ensure the stability of the analysis, and it defines the so-called explicit and implicit approaches of the model. The evaluation of the α parameter, on the other hand, focuses on enabling an effective numerical dissipative algorithm, aiming to eliminate the influence of spurious modes and to reduce amplitude decay errors; it defines the so-called dissipative and non-dissipative elements of the model, which are relabeled at each time step of the analysis. The proposed adaptive strategy is non-iterative, and the values for the time integrators are simply and directly computed taking into account just the physical/geometrical proprieties of the finite elements of the spatial discretization, the adopted time-step, and local previous time-step results. In addition, the proposed technique is only based on single-step relations involving two variables: the incognita field and its first time derivative. Thus, just a single set of equations has to be dealt with within a time-step (which becomes trivial if explicit analyses are considered), and the resulting method stands as truly self-starting, eliminating any kind of cumbersome initial procedure, such as the computation of initial second time derivative values and/or the computation of multistep initial values.

Since the adopted time-marching formulation enables explicit or implicit analyses to be carried out, the acoustic fluid subdomains of the model may consider an explicit formulation, allowing the coupled governing system of equations to get uncoupled along time. Thus, each subdomain of the coupled model can be analysed separately (as a staggered uncoupled model), rendering a very efficient approach. In the present work, two time domain approaches are discussed, taking into account the solution of the coupled fluid-solid model. In the first approach, the acoustic and the elastodynamic subdomains of the model consider an explicit time domain formulation, rendering an explicit-explicit technique. In the second approach, the acoustic and the elastodynamic subdomains of the model consider an explicit and an implicit time domain formulation, respectively, rendering an explicit-implicit technique. In both cases, the effectiveness of the proposed techniques is remarkable.

The manuscript is organized as follows: first (Section 2), the governing equations for the acoustic fluid and elastodynamic solid subdomains are presented, as well as their coupling equations. In the sequence, the spatial and temporal discretizations for the different subdomains are briefly described (Section 3) and the proposed algorithm for the coupled model solution is discussed (Section 4). At the end of the paper (Section 5), numerical results are presented, illustrating the accuracy and potentialities of the proposed methodologies.

2. Governing equations

In the present section, acoustic and elastic wave equations are briefly discussed. These wave propagation models are used to describe the fluid and solid subdomains of the coupled problem that is focused here. At the end of the section, the basic equations related to the coupling of the acoustic and elastodynamic subdomains are presented.

2.1. Acoustic subdomains

The acoustic wave equation is given by:

$$(\kappa p_{,i})_{,i} - \rho\ddot{p} - \varsigma\dot{p} + a = 0 \tag{1}$$

where $p(\mathbf{x},t)$ stands for the hydrodynamic pressure distribution and $a(\mathbf{x},t)$ for body source terms. Inferior commas (indicial notation is adopted) and over dots indicate partial space ($p_{,i} = \partial p/\partial x_i$) and time ($\dot{p} = \partial p/\partial t$) derivatives, respectively. $\rho(\mathbf{x})$ stands for the mass density, $\kappa(\mathbf{x})$ is the bulk modulus of the medium, and $\varsigma(\mathbf{x})$ represents the viscous damping parameter. In homogeneous media, ρ and κ are constant and the classical wave equation can be written, with $\varsigma = 0$, as:

$$p_{,ii} - (1/c^2)\ddot{p} + a/\kappa = 0 \tag{2}$$

where $c = \sqrt{\kappa/\rho}$ is the wave propagation speed. The boundary and initial conditions of the problem are given by:

(i) Boundary conditions ($t > 0$, $\mathbf{x} \in \Gamma$ where $\Gamma = \Gamma_1 \cup \Gamma_2$)

$$p(\mathbf{x},t) = \bar{p}(\mathbf{x},t) \quad \text{for } \mathbf{x} \in \Gamma_1 \tag{3a}$$

$$q(\mathbf{x},t) = p_{,j}(\mathbf{x},t)n_j(\mathbf{x}) = \bar{q}(\mathbf{x},t) \quad \text{for } \mathbf{x} \in \Gamma_2 \tag{3b}$$

(ii) Initial conditions ($t = 0$, $\mathbf{x} \in \Gamma \cup \Omega$):

$$p(\mathbf{x},0) = \bar{p}_0(\mathbf{x}) \tag{4a}$$

$$\dot{p}(\mathbf{x},0) = \dot{\bar{p}}_0(\mathbf{x}) \tag{4b}$$

where the prescribed values are indicated by overbars and q represents the flux along the boundary whose unit outward normal vector components are represented by n_j. The boundary of the model is denoted by Γ ($\Gamma_1 \cup \Gamma_2 = \Gamma$ and $\Gamma_1 \cap \Gamma_2 = \varnothing$) and the domain by Ω.

2.2. Elastodynamic subdomains

The elastic wave equation for homogenous media is given by:

$$(c_d^2 - c_s^2)u_{j,ji} + c_s^2 u_{i,jj} - \ddot{u}_i - \varsigma'\dot{u}_i + b_i = 0 \tag{5}$$

where $u_i(\mathbf{x},t)$ and $b_i(\mathbf{x},t)$ stand for displacement and body force distribution components, respectively. c_d is the dilatational wave velocity and c_s is the shear wave velocity, which are given by: $c_d^2 = (\lambda + 2\mu)/\rho$ and $c_s^2 = \mu/\rho$, where ρ is the mass density and λ and μ are the Lamé's constants of the medium. The viscous damping term is defined by $\varsigma' = \varsigma/\rho$. Eq. (5) can be obtained from the combination of the following basic mechanical equations (proper to model heterogeneous media):

$$\sigma_{ij,j} - \rho\ddot{u}_i - \varsigma\dot{u}_i + \rho b_i = 0 \tag{6a}$$

$$\sigma_{ij} = \lambda\delta_{ij}\varepsilon_{kk} + 2\mu\varepsilon_{ij} \tag{6b}$$

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{6c}$$

where $\sigma_{ij}(\mathbf{x},t)$ and $\varepsilon_{ij}(\mathbf{x},t)$ are, respectively, stress and strain tensor components and δ_{ij} is the Kronecker delta ($\delta_{ij} = 1$, for $i = j$ and $\delta_{ij} = 0$, for $i \neq j$). Eq. (6a) is the momentum equilibrium equation; Eq. (6b) represents the constitutive law of the model; and Eq. (6c) stands for kinematical relations. The boundary and initial conditions of the elastodynamic problem are given by:

(i) Boundary conditions ($t > 0$, $\mathbf{x} \in \Gamma$ where $\Gamma = \Gamma_1 \cup \Gamma_2$)

$$u_i(\mathbf{x},t) = \bar{u}_i(\mathbf{x},t) \quad \text{for } \mathbf{x} \in \Gamma_1 \tag{7a}$$

$$\tau_i(\mathbf{x},t) = \sigma_{ij}(\mathbf{x},t)n_j(\mathbf{x}) = \bar{\tau}_i(\mathbf{x},t) \quad \text{for } \mathbf{x} \in \Gamma_2 \tag{7b}$$

(ii) Initial conditions ($t = 0$, $\mathbf{x} \in \Gamma \cup \Omega$)

$$u_i(\mathbf{x},0) = \bar{u}_{i0}(\mathbf{x}) \tag{8a}$$

$$\dot{u}_i(\mathbf{x},0) = \dot{\bar{u}}_{i0}(\mathbf{x}) \tag{8b}$$

where once again, the prescribed values are indicated by overbars and $\tau_i(\mathbf{x},t)$ denotes the traction vector along the boundary (n_j, as previously indicated, stands for the components of the unit outward normal vector).