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Nonlinear kinematics Reissner's beam with combined hardening/softening elastoplasticity

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ABSTRACT

In this work, we present geometrically non-linear beam finite element with embedded discontinuity which can represent elastoplastic constitutive behavior with both hardening and softening response. The constitutive equations are presented in rate form by using the multiplicative decomposition of deformation gradient. Formulation of elastoplastic response is presented in terms of stress resultants including the interaction between axial force, shear force and bending moment appropriate for metallic materials. The softening response is used to model the failure in connections, introducing displacement field discontinuity or a rotational hinge. The hinges or displacement discontinuity are presented in the framework of incompatible modes that can handle three different failure modes dealing with bending, shearing or axial deformation. With several numerical simulations, the FEM implementation is proven very robust for solving the problems of practical interest, such as push-over analysis.

1. Introduction

The model capable of predicting the complete failure (collapse) of a frame structure is very important in the limit load design. Typical application is push-over analysis used in earthquake engineering; a nonlinear static analysis of a building structure subjected to an equivalent static load that is pushing a structure towards the limit capacity. This type of the analysis was developed in work [1] as incompatible modes in the small displacement framework. During push-over analysis of a structure, there are hinges that develop, in a step-by-step manner, leading to the failing mechanism. In structural analysis those hinges can be included by using static condensation method [7]. The incompatible mode method is more robust, while the static condensation method is more efficient. For improved prediction, it is necessary [1] to include geometric nonlinearities of the second order, indicating the need for improvement.

The truly large kinematics of steel frame structures combined with elastoplastic hardening/softening is the main novelty of this work. The ductile material like steel can handle large displacements and deformation of a structure during the limit load analysis. The geometrically exact beam with nonlinear kinematics and nonlinear constitutive behavior should be capable of following response of a structure to the complete failure (collapse). In this work, we propose elastoplastic beam element in geometrically nonlinear regime [4] that can handle softening response, which is included in the framework of incompatible modes.

In the formulation of the proposed beam element we use, as the starting point, the previous works [4,11]. The proposed beam element includes nonlinear kinematics and nonlinear constitutive response. The constitutive behavior is defined as plasticity with linear hardening that includes interaction between axial force, shear force and bending moment. The evolution equations for internal variables are developed in rate form, imposing the need to employ a numerical time integration scheme, -here chosen as the backward Euler scheme.

The main novelty concerns the beam model's ability to reach the ultimate capacity of a cross section, activating one of three failure modes, which represent non-linear softening response in either bending moment, shear or axial force. These failure modes are handled by field discontinuity as incompatible modes, see [5]. In this work, we presume that only one softening failure mechanism can be activated at the time. The outline of the paper is as follows.

In the next section, we present the main ingredients of the geometrically exact beam with the elastoplastic constitutive response. The interaction between axial force, shear force and bending moment is taken in the elastoplastic regime, while the axial response remains elastic. Section 2 presents corresponding kinematic enhancement in terms of "discontinuity" or "jump" in

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the displacement field or the rotational field depending upon the activated failure mode. The enhancement is included as an incompatible mode in the geometrically nonlinear framework. Section 3 deals the FEM implementation, while Section 4 presents the results of several numerical simulations. Section 5 contains the conclusions.

2. Reissner's beam with non-linear kinematics

In this section, we give a detailed formulation of the two-dimensional beam in the framework of large displacement and large elastoplastic strains. The formulation of Reissner's beam [10] kinematics equations employs rotated strain measure. The linearization of these strain measures reduce the strains of the Timoshenko beam [4,9]. The plastic strains corresponding to stress resultant follow from yield criterion introducing the interaction between axial force, shear force and bending moment. The equations are expressed in rate form [11]. The consistent linearization of the weak form of equilibrium equations provides tangent stiffness matrix, for both material and geometric part.

Providing the beam element with the embedded discontinuity within the framework of a large displacement is needed for modeling softening phase. The later can concern the failure process in the connections, modeling separately the failure in bending, in shearing or in axial force. The multiplicative decomposition of the deformation gradient into regular and irregular parts corresponds to the additive decomposition of the rotated strain measure proposed by Reissner [10]. Moreover, the weak form of equilibrium equation has to be recast within the framework of incompatible modes [5], which allows handling of the embedded discontinuity calculation at the element level.

2.1. Geometrically nonlinear kinematics

In the framework of large displacement gradient theory, the position vector in deformed configuration can be written as

$$\boldsymbol{\varphi} := \boldsymbol{\varphi_0} + \zeta \mathbf{t} = \begin{pmatrix} x+u \\ y+v \end{pmatrix} + \zeta \begin{pmatrix} -\sin\psi \\ \cos\psi \end{pmatrix} \tag{1}$$

where x and y are coordinates in the reference configuration, u and v are displacement components in the global coordinate system, ζ is the coordinate along the normal to the beam axis in the reference configuration and ψ is the rotation. The corresponding form of the deformation gradient $\mathbf{F}$ can be split into displacement part $\mathbf{F}_{u,v}$ and rotation part $\mathbf{F}_{\psi}$ as:

$$\mathbf{F} := \nabla\boldsymbol{\varphi} = \underbrace{\begin{bmatrix} 1+\dfrac{du}{dx} & 0 \\ \dfrac{dv}{dx} & 0 \end{bmatrix}}_{\mathbf{F}_{u,v}=\mathbf{I}+\nabla\mathbf{u}} + \underbrace{\begin{bmatrix} -\zeta\dfrac{d\psi}{dx}\cos\psi & -\sin\psi \\ -\zeta\dfrac{d\psi}{dx}\sin\psi & \cos\psi \end{bmatrix}}_{\mathbf{F}_{\psi}=\mathbf{I}+\nabla\boldsymbol{\psi}} \tag{2}$$

The failure mode in connection can be represented by jump in displacement components u, v and in the rotation ψ, with the corresponding kinematic enhancement in terms of the "discontinuity". In the finite deformation framework, such a displacement discontinuity has to be introduced in deformed configuration [4]. This splits displacement field into the regular part $(\bar{\bullet})$ and the "enhanced" part $(\bar{\bar{\bullet}})$ representing the corresponding displacement or rotation "jump". By introducing $\delta_{\bar{x}}$ as the Dirac function where the jump occurs, the additive decomposition of displacements and rotation gradient fields can be written as:

$$\begin{aligned}
u(x,t) &= \tilde{\bar{u}}(x,t) + (N_a(x)+H(x))\bar{\bar{u}}(t) \rightarrow \frac{\partial u}{\partial x} = \frac{\partial \tilde{\bar{u}}}{\partial x} + G_a(x)\bar{\bar{u}} + \delta_{\bar{x}}\bar{\bar{u}} = \frac{\partial \bar{u}}{\partial x} + \delta_{\bar{x}}\bar{\bar{u}} \\
v(x,t) &= \tilde{\bar{v}}(x,t) + (N_a(x)+H(x))\bar{\bar{v}}(t) \rightarrow \frac{\partial v}{\partial x} = \frac{\partial \tilde{\bar{v}}}{\partial x} + G_a(x)\bar{\bar{v}} + \delta_{\bar{x}}\bar{\bar{v}} = \frac{\partial \bar{v}}{\partial x} + \delta_{\bar{x}}\bar{\bar{v}} \\
\psi(x,t) &= \tilde{\bar{\psi}}(x,t) + (N_a(x)+H(x))\bar{\bar{\psi}}(t) \rightarrow \frac{\partial \psi}{\partial x} = \frac{\partial \tilde{\bar{\psi}}}{\partial x} + G_a(x)\bar{\bar{\psi}} + \delta_{\bar{x}}\bar{\bar{\psi}} = \frac{\partial \bar{\psi}}{\partial x} + \delta_{\bar{x}}\bar{\bar{\psi}}
\end{aligned} \tag{3}$$

where $N_a(x)$ is interpolation function, $H(x)$ is Heaviside function and $G_a(x)$ is the first derivative of the interpolation function $N_a(x)$. By using last result (3) we can write the deformation gradient for both the displacement and the rotation fields, in terms of the multiplicative decomposition of:

$$\begin{aligned}
\mathbf{F} &= \mathbf{I} + \nabla\bar{\mathbf{u}} + \delta_{\bar{x}}\nabla\bar{\bar{\mathbf{u}}} + \mathbf{I} + \nabla\bar{\psi} + \delta_{\bar{x}}\nabla\bar{\bar{\psi}} \\
&= (\mathbf{I}+\nabla\bar{\mathbf{u}})\left(\mathbf{I} + \delta_{\bar{x}}\frac{\nabla\bar{\bar{\mathbf{u}}}}{I+\nabla\bar{\mathbf{u}}}\right) + (\mathbf{I}+\nabla\bar{\psi})\left(\mathbf{I} + \delta_{\bar{x}}\frac{\nabla\bar{\bar{\psi}}}{I+\nabla\bar{\psi}}\right) \\
&= \bar{\mathbf{F}}_{\mathbf{u},\mathbf{v}}\bar{\bar{\mathbf{F}}}_{\mathbf{u},\mathbf{v}} + \bar{\mathbf{F}}_{\psi}\bar{\bar{\mathbf{F}}}_{\psi}
\end{aligned} \tag{4}$$

From the polar decomposition of the deformation gradient $\mathbf{F}$, into rotation $\mathbf{R}$ and stretch $\mathbf{U}$, we define the rotated strain measure $\mathbf{H}$:

$$\mathbf{F} = \mathbf{R}\mathbf{U} \rightarrow \mathbf{U} = \mathbf{R}^T\mathbf{F}, \ \mathbf{R} = \begin{bmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{bmatrix} \rightarrow \mathbf{H} = \mathbf{U} - \mathbf{I} \tag{5}$$

where $\mathbf{I}$ is identity tensor. With the results (4) and (5), we can obtain the corresponding additive decomposition of the stretch tensor:

$$\mathbf{U} = \mathbf{R}^{\mathrm{T}}\left(\mathbf{I} + \nabla\bar{\mathbf{u}} + \delta_{\bar{x}}\nabla\bar{\bar{\mathbf{u}}}\right) + \mathbf{R}^{\mathrm{T}}\left(\mathbf{I} + \nabla\bar{\psi} + \delta_{\bar{x}}\nabla\bar{\bar{\psi}}\right) = \underbrace{\bar{\mathbf{U}}^{u,v} + \delta_{\bar{x}}\bar{\bar{\mathbf{U}}}^{u,v}}_{\mathbf{U}^{u,v}} + \underbrace{\bar{\mathbf{U}}^{\psi} + \delta_{\bar{x}}\bar{\bar{\mathbf{U}}}^{\psi}}_{\mathbf{U}^{\psi}} \tag{6}$$

where

$$\bar{\mathbf{U}}^{\mathbf{u},\mathbf{v}} = \begin{bmatrix} (1+\frac{\partial\bar{u}}{\partial x})\cos\psi + \frac{\partial\bar{v}}{\partial x}\sin\psi & 0 \\ -(1+\frac{\partial\bar{u}}{\partial x})\sin\psi + \frac{\partial\bar{v}}{\partial x}\cos\psi & 1 \end{bmatrix}; \ \bar{\bar{\mathbf{U}}}^{\mathbf{u},\mathbf{v}} = \begin{bmatrix} \bar{\bar{u}}\cos\psi + \bar{\bar{v}}\sin\psi & 0 \\ -\bar{\bar{u}}\sin\psi + \bar{\bar{v}}\cos\psi & 0 \end{bmatrix}\delta_{\bar{\mathbf{x}}}$$

$$\bar{\mathbf{U}}^{\psi} = \begin{bmatrix} -\zeta\frac{\partial\bar{\psi}}{\partial x} & 0 \\ 0 & 1 \end{bmatrix}; \ \bar{\bar{\mathbf{U}}}^{\mathbf{u},\mathbf{v}} = \begin{bmatrix} -\zeta\bar{\bar{\psi}} & 0 \\ 0 & 0 \end{bmatrix}\delta_{\bar{\mathbf{x}}}$$

Finally, we can write the internal virtual work in an alternative form that is more in line with the corresponding 3D representations [4]

$$\int_L \int_A \hat{\mathbf{F}} \cdot \mathbf{P} dA dx = \int_L \int_A \hat{\mathbf{H}} \cdot \mathbf{T} dA dx \tag{7}$$

where $\hat{\mathbf{F}}$ is variation of the deformation gradient, $\mathbf{P}$ is first Piola-Kirchhoff stress. In last Eq. (7), we used the following result for Biot stress tensor $\mathbf{T}$ and corresponding rotated strain measures $\mathbf{H}$ and their variations $\hat{\mathbf{H}}$:

$$\mathbf{T} = \mathbf{R}^{\mathbf{T}}\mathbf{P} \rightarrow \begin{pmatrix} T^{11} \\ T^{21} \end{pmatrix} = \mathbf{R}^{\mathbf{T}}\begin{pmatrix} P^{11} \\ P^{21} \end{pmatrix} \tag{8}$$

2.2. Constitutive model and its rate form

In the elastic regime the simplest set of constitutive equations for finite strain beam is chosen in terms of Biot stress resultants and rotated strain measure:

$$\mathbf{T} = \mathbf{C}^e\mathbf{H} \tag{9}$$