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Higher-order thin-walled beam analysis for axially varying generally shaped cross sections with straight cross-section edges

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ABSTRACT

A higher-order beam theory is proposed for the analysis of a thin-walled beam with a generally shaped cross section, which consists of straight cross-section edges and is non-uniform along the axial direction. To derive cross-sectional shape functions for the higher-order deformation modes, a new approach is introduced using a set of beam frame models. The distortions with inextensional cross-sectional walls are determined by solving an eigenvalue problem of a beam frame model under inextensional wall constraints. Subsequently, the distortions with extensional cross-sectional walls are evaluated by considering orthogonality with respect to the inextensional distortions. Moreover, the extensional distortions due to the Poisson effect, which is generated due to the uniform axial strain of the rigid-body cross-sectional deformations, are considered. Warpings induced by the inextensional and extensional distortions are consistently defined based on the orders of the tangential displacements of their corresponding distortions. To deal with the varying cross section, three-dimensional displacements at an arbitrary point are interpolated using those at the cross sections of the nodes, where the beam frame analyses are performed. The proposed method is validated by performing static and vibration analyses of beams with varying single- and multi-cell cross sections.

1. Introduction

Frame structures, such as automotive bodies, comprise beams with generally shaped cross sections that vary in the axial direction. Although the static and dynamic behavior of a thin-walled beam structure can be accurately evaluated using shell elements, interpreting the structural response is difficult from the viewpoint of the cross-sectional property of the beam. The use of beam elements for the analysis of thin-walled beams with varying cross section, however, does not yield sufficiently accurate results for implementation in a design process. To solve this problem, higher-order cross-sectional deformations need to be considered, and the effect of cross-sectional variation must be appropriately reflected while formulating the beam elements.

In a higher-order beam theory, higher-order cross-sectional deformations are employed as field variables; three-dimensional displacements at a general point on a beam are approximated using cross-sectional shape functions and one-dimensional deformation measures (or field variables). By doing so, three-dimensional elasticity equations can be reduced to a set of one-dimensional governing equations, whose coefficients are calculated by integrating the products of the cross-sectional shape functions or their derivatives. Since the early studies by Vlasov [1] and von Kármán and Christensen [2] on warpings in open and closed thin-walled cross sections, respectively, many studies on higher-order beam theories have been conducted. For example, Boswell and Zhang [3] analyzed straight and curved girder bridges with symmetric single- and multi-cell cross sections using three additional higher-order cross-sectional modes: torsional warping, distortion, and distortional warping. In their approach, a zero shear strain assumption on the midline of the cross-sectional walls, proposed by Vlasov [1], was employed to derive the warpings due to torsion or distortion. The use of distortions for the analysis of a symmetrical cross section can be also found in the studies by Kermani and Waldron [4] and Kim and Kim [5], Kim and Kim [6]. Kim and Kim [5], Kim and Kim [6] analytically derived the shape functions of the torsional and bending distortions for a rectangular cross section by assuming constant tangential displacements; moreover, they employed kinematic and moment continuity conditions for the corners of the cross section. In these approaches, the main objective was to analytically derive the shape functions of a single distortion and its associated warping, which are

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expected when straight and curved beams undergo torsion and in-plane bending, respectively. Hence, their effectiveness was limited because of the geometry of the cross section as well as loading conditions.

Recent studies on higher-order beams employ multiple warpings and distortions as out-of-plane and in-plane cross-sectional deformations, respectively, to deal with any given geometry of a cross section and general loading conditions. Ferradi and Cespedes [7] derived distortions of a cross section by using the eigenmodes of a two-dimensional cross-sectional model, which was discretized with triangular solid elements or beam elements. For each distortion mode, they calculated multiple warping modes using an iterative scheme. Vieira et al. [8] expressed beam-governing equations as a quartic eigenvalue problem, from which they derived not only the non-vanishing deformations corresponding to the zero eigenvalues, but also the warping and distortional modes corresponding to the nonzero eigenvalues. Moreover, they simplified a generalized eigenvalue problem to a linear form by assuming no in-plane higher-order deformations [9]. The generalized beam theory (GBT) introduced by Schardt [10] was used to calculate higher-order cross-sectional deformations based on the assumptions of inextensible walls and zero shear strain on the midline of walls. In GBT, using the piecewise linear-warping modes, rigid, distortional, and local modes are derived as an orthogonal set of the in-plane deformations [11,12]. Gonçalves et al. [13], Piccardo et al. [15], and Bebiano et al. [14] relaxed the kinematic assumptions of the inextensible walls and zero shear strain so that the extensional distortion modes and shear modes are introduced. Instead of calculating the higher-order modes for a given cross section, displacement fields can be developed through Taylor series as a unified formulation. Carrera et al. [16] proposed to refine the three-dimensional displacements at an arbitrary point on a cross section using Taylor-type polynomial expansions with respect to the cross-sectional coordinates. They applied the unified formulation to a cross section with a general geometry for static, dynamic, and buckling problems of isotropic or composite beams [17–19]. The beam-governing equations can also be derived by applying the variational asymptotic method to the energy functional [20,21]. The effect of the warping was derived using the variational asymptotic method for problems regarding geometrically nonlinear beams such as initially twisted and curved composite beams [20].

In this study, static and vibration problems of a straight or curved thin-walled beam with a generally shaped cross section that varies in the axial direction are solved. To this end, the static analysis performed by Choi et al. [22] for quadrilateral cross-sectional problems is extended. They showed that the geometric complexity of a varying cross section generates coupling between the deformation measures; hence, the order of the cross-sectional deformation modes is higher than the case wherein the cross section is uniform. Choi et al. [22] employed distortion modes with extensional walls as well as those with inextensional walls. In their approach, distortions were calculated by solving the eigenvalue problem of a beam frame model, obtained from the study by Jang et al. [23]. Some of the extensional distortion modes, referred to as the Poisson distortions, can be analytically derived by solving equilibrium equations of plane stress conditions, assuming uniform strains in the axial direction of a beam for three rigid-body cross-sectional deformations: axial translation via extension/compression and two rotations via bending. Hence, the number of Poisson distortions is three. For each distortion of a quadrilateral cross section, a corresponding warping is also analytically derived in an integration form. Unfortunately, the analytical method, proposed by Choi et al. [22], cannot be employed for a cross section with a general shape because of the lack of constraint equations; the number of unknowns resulting from the integration of the equilibrium equation is greater than the number of kinematic continuity conditions, moment continuity equations, and orthogonality conditions.

To consider a general geometry of a thin-walled cross section, a set of beam frame models are introduced in this study. For the distortion modes, a beam frame model discretized using the Euler beam elements are employed, from which distortions are selected as the lowest eigenmodes. For the warping modes, a beam frame is modeled differently depending on whether the corresponding distortions are inextensional or extensional. As warping is obtained from the equilibrium equation by double integrating the derivative of the tangential displacement of its corresponding distortion [22], the order of the warping should be set higher than that of the tangential displacement of the distortion. Hence, linear and quadratic beam frame models are separately employed to calculate warpings induced by inextensional and extensional distortion modes, respectively. Because of the relation between the order of the tangential displacement of distortion and the corresponding warping, each distortion needs be identified as either an inextensional mode or an extensional mode. In this study, first, the inextensional distortion modes are calculated by setting inextensional wall constraints; subsequently, the extensional distortion modes are calculated by considering orthogonality with respect to the inextensional distortions. The edgewise linear shapes in the tangential directions are allowed for the extensional distortion modes so that the corresponding warpings are consistently obtained using a beam frame model with edgewise quadratic shape functions. Moreover, the warping modes for the inextensional distortions are obtained as eigenmodes of a linear beam frame. Similarly, Piccardo et al. [15] attempted to separate the inextensional and extensional distortion modes. However, in this study, no constraint is set on the tangential deformation of the extensional distortions, and all the warping modes are obtained by solving the eigenvalue problem of a beam frame with linear shape functions.

By interpolating the three-dimensional displacements at the cross sections of the nodes, where the beam frame analyses are performed, three-dimensional displacements at an arbitrary point of a beam element can be obtained. The strains and stresses expressed in terms of the local coordinates on the wall of a beam element are calculated using coordinate transformations, from which stiffness and mass matrices can be derived using the principle of virtual displacements. To validate the proposed method, straight and curved beam problems with varying closed cross section are solved. The effect of the higher-order modes on the accuracy of the solution is investigated using different sets of degrees of freedom. A twisted open cross-sectional problem and a multi-cell cross-sectional problem are also solved. The results of the vibration analyses are presented for the beams with single- and multi-cell cross sections.

2. Cross-sectional shape functions for general cross sections

2.1. Displacements on the midline of cross section

Fig. 1 illustrates the generally shaped cross section of a thin-walled beam, which comprises finite number of straight edges (or walls). The three-dimensional displacements on the midline of the cross-sectional walls are expressed as follows.

$$u_p(s,z) = \sum_{i=1}^{N_\psi} \psi_p^i(s) d_i(z) = \boldsymbol{\psi}_p \mathbf{d}, \quad (p = n, s, z), \tag{1}$$

where s and n are the tangential and outward normal coordinates on the cross-sectional contour, respectively, and z is the axial coordinate of the beam (see Fig. 1.) In Fig. 1, note that s and n are edgewise defined. In Eq. (1), $\psi_p^i(s)$ denotes the shape functions for the