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Structural identification using a nonlinear constraint satisfaction processor with interval arithmetic and contractor programming

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Structural identification through finite element model updating has gained increased importance as an applied experimental technique for performance-based structural assessment and health monitoring. However, practical challenges associated with computability, feasibility, and uniqueness present in the structured nonlinear inverse eigenvalue problem develop as a result of the necessary use of partially described and incompletely measured mode shapes. As an alternative to direct methods and optimization-based approaches, this paper proposes a new paradigm for model updating that is based on formulating the structured inverse eigenvalue problem as a Constraint Satisfaction Problem. Interval arithmetic and contractor programming are introduced as a means for generating feasible solutions to a structured inverse eigenvalue problem within a bounded parameter search space. This framework offers the ability to solve under-determined and non-unique inverse problems as well as accommodate measurement uncertainty through relaxation of constraint equations. These abilities address key challenges in quantifying uncertainty in parameter estimates obtained through structural identification and enable the exploration of alternative solutions to the global minimum that may better reflect the true physical properties of the structure. These capabilities are first demonstrated using synthetic data from a numerical mass-spring model and then extended to experimental data from a laboratory shear building model. Lastly, the methodology is contrasted with probabilistic model updating to highlight the advantages and unique capabilities offered by the methodology in addressing the effects of measurement uncertainty on the parameter estimation.

1. Introduction

Over the past several decades, numerous techniques have been devised to develop updated structural stiffness and mass matrices from modal parameter estimates obtained from either experimental or operational modal analysis. Within structural identification, the properties of the updated model are used to infer the behavior and performance of the structure to inform decision-making [1]. Consequently, instilling confidence that parameter assignments in the updated model closely reflect physical reality is critical to the use of structural identification for applications in performance-based civil engineering and structural health monitoring. Likewise, understanding the uncertainty in the parameter estimates in the presence of measurement noise and potential ill-conditioning of the inverse problem is necessary to provide reliable and actionable information.

Traditionally, the finite element model updating problem has been framed using the generalized eigenvalue problem for undamped multiple degree of freedom linear systems:

$$K\Phi = M\Phi\Omega^2 \tag{1}$$

where M and K are the mass and stiffness matrices, Ω^2 is a diagonal matrix containing the eigenvalues (ω_n^2, where ω_n are the undamped natural frequencies) on the diagonal, and Φ is the matrix containing the corresponding eigenvectors, or mode shapes, as columns of the matrix. This generalized eigenvalue problem has been adopted for the majority of structural identification applications since the finite element method can be used to readily construct the mass and stiffness matrices, while experimentally measured relative damping factors can be used to construct the corresponding damping matrix for the system. The general objective of model updating is to modify the stiffness and mass matrices of an analytical model of the structure such that the eigenproperties of the model obtain the best possible match to experimentally measured values, while preserving physical meaning and structural constraints in the matrices.

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Although this objective is easily described, there are many practical challenges that arise from the nature of vibration testing and analytical modeling that are ubiquitous to all methods employed for model updating. Namely:

- The practical measurement bandwidth of vibration transducers is limited, which effectively limits the measurement of the eigenvalues and eigenvectors of the system under test to typically a small subset of all of those that would exist in the corresponding analytical model. This case is commonly referred to as having partially described eigeninformation pairs [2].
- It is impractical to completely measure all corresponding degrees of freedom in the analytical model, so the experimental mode shapes are incomplete measurements of the eigenvector. For models of even modest size structures, the number of sensors required to measure every degree of freedom in a sufficiently discretized finite element model is generally prohibitive. Furthermore, the direct measurement of some rows of the eigenvector, such as those associated with the rotational degrees of freedom, is not even possible with conventional transducers [3].
- Noise and uncertainty in the measurements, as well as assumptions inherent to system identification algorithms, yield mode shapes that no longer satisfy orthogonality relationships and are unlikely to satisfy equality constraints. Likewise, discretization errors and idealizations inherent to the model, such as element type and mesh connectivity, are not explicitly corrected in conventional finite element model updating schemes [4].

Currently, the two most prevalent techniques applied for structural identification are deterministic and probabilistic methods of finite element model updating. Deterministic methods seek to identify optimal assignments for a set of uncertain parameters in the model by minimizing the residuals between measured and estimated modal parameters by application of various optimization techniques [5–7]. While these techniques have been applied for structural identification of several full-scale structures [8,9], their application is generally plagued by issues associated with computational speed, solution uniqueness, ill-conditioning, and parameter selection and sensitivity [10]. Furthermore, finite element model updating problems suffer from the underlying issue that the global minimum may not necessarily reflect the best match to the physical reality due to uncorrected errors arising from idealization and discretization in the model and uncertainties in the measurements [4]. These challenges have given rise to probabilistic finite element model updating approaches [11], which incorporate uncertainties in the model and the measurements to identify the most probable solutions using statistical methods. However, these probabilistic methods require an assumed probability density function for uncertain variables and often require computationally expensive simulations to arrive at the solution [7]. Lastly, it should be noted that direct methods for solving inverse eigenvalue problems with incompletely measured mode shapes through solution of a descriptor Sylvester equation have recently emerged [12,13], however their application has yet to demonstrate the capability to preserve the connectivity structure of the matrices.

Although probabilistic approaches have been the most prevalently used to address uncertainties in structural models, interval methods provide an alternative approach that may offer computational advantages over probabilistic techniques [14,15]. Recently, interval-based model updating strategies have been proposed for handling inherent uncertainties in the experimental modal parameter estimates and the finite element model [16]. The most closely related work to the current study leverages interval global optimization to arrive at solutions to the model updating problem [17]. However, the technique proposed in this prior work is formulated on an inclusion set for the eigenvalues of an interval stiffness matrix and employs a simple branch and bound algorithm to minimize an objective function rather than solve a constraint satisfaction problem. An extension of this work incorporated interval eigenvectors in the optimization through an acceptance criteria based on the modal assurance criterion [18]. It should also be noted that the term "interval model updating" has also been recently used to described the application of interval arithmetic techniques in the estimation of parameter variability [19,20]. However, these approaches address the field of stochastic model updating [21], wherein the objective is to characterize the variabilities across a number of experimental tests performed on nominally identical structures. While the constraint satisfaction formulation proposed in the current work may have extensions to stochastic model updating, the subject of this current paper is on the class of model updating applications where measurements obtained from a single structure are used to calibrate a numerical model.

This paper presents a novel formulation of the finite element model updating problem as a constraint satisfaction problem and explores the use of a nonlinear constraint satisfaction processor with interval arithmetic and contractor programming to yield estimates of uncertain model parameters and unmeasured components of the eigenvectors. The method is shown to be capable of delivering a complete set of feasible solutions to the structured inverse eigenvalue problem with partially described and incompletely measured eigeninformation pairs from either synthetic or experimental data. Furthermore, the approach is successfully demonstrated on ill-posed problems with multiple solutions to illustrate its capability for addressing this challenge as well as providing a foundation to introduce practitioner heuristics into the identification of physically plausible solution sets.

2. Nonlinear constraint satisfaction with contractor programming and interval analysis

In the domain of engineering sciences, many applications require finding all possible and potentially isolated solutions satisfiable to a set of constraints over real numbers. The system of equations may be non-polynomial and the computational complexity to solve such systems is NP-hard. This set of problems are called Constraint Satisfaction Problems (CSPs) [22–24]. The approach explored in this paper for the solution of partially described and incompletely measured inverse eigenvalue problems relies on framing the structured inverse eigenvalue problem as a nonlinear CSP. By developing the model updating problem in this framework, unique capabilities for addressing challenges related to ill-posedness and ill-conditioning are revealed, as detailed in later sections. The following discussion provides some details on the fundamentals of CSPs, interval analysis, and contractor programming that are essential for the understanding of the rest of the paper.

Fundamentally, a CSP can be defined as a *4-tuple* $< V, D, C, L >$ where:

- $V = \{v_1, v_2, \ldots, v_n\}$ is a set of variables,
- $D = \{d_1, d_2, \ldots, d_n\}$ is a set of domains for prospective variables,
- $C = \{c_1, c_2, \ldots, c_n\}$ is a set of constraints over the variables,
- L is a set of labels that map constraints to the variables and corresponding domains, formally: $L : C_i \rightarrow (v_i, d_i)$.

Each variable, v_i, can assume any real value in the corresponding non-empty domain d_i. The constraint $c_i \in C$ is defined over a pair (v_i, d_i) through a label function $l \subset L$. In the process of finding a satisfiable solution to a CSP, different values are assigned to the