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The safety of masonry arches with uncertain geometry

N. Cavalagli, V. Gusella, L. Severini *

Department of Civil and Environmental Engineering, University of Perugia, Italy

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ABSTRACT

This paper aims to evaluate the effect of the geometrical uncertainties on the collapse condition of the circular masonry arch in presence of horizontal actions. Adopting Heyman's hypotheses about the material, a limit analysis based procedure has been developed in order to evaluate the horizontal loads multiplier, taking into account the uncertainties related to the imprecisions of construction, the shape defects of the voussoirs or the deterioration level. The collapse state has been determined in terms of horizontal loads multiplier, whose statistical moments up to second order and probability density functions have been evaluated versus a stereometry parameter. The comparison between the obtained results and those related to the nominal geometry highlighted that the uncertainties effects could reduce significantly the nominal bearing capacity of the structure. Within this context, a safety factor, which takes into account such effects, is introduced.

1. Introduction

The cultural and architectural heritage in seismic areas often shows a high vulnerability, requiring a special attention in order to ensure its protection and conservation. Many ancient structures are characterized by the presence of the masonry arch as bearing element, since it was one of the first structural solutions to be thought in order to carry loads. Although the arches and vaults have already been known, the credit to understand the potential of these elements goes to the Romans. The architect Vitruvius, in his famous book De Architectura, proves to have a deep knowledge of the thrust exercised by arches or vaults. The Romans consolidated the constructive practice by using the arch for the bridges and aqueducts construction, determining in this way the development of the infrastructure network. Starting from the Middle Age and passing through the Renaissance, the construction technique of the masonry arches led to architectural masterpieces that were designed according to stability criteria, although at the time a clear static or mechanical justification had not yet been provided. When designing a masonry structure, both the medieval masters and then the architects of Humanism and Renaissance referred to their system of practical rules, jealously guarded. Some of them can be traced in ancient treatises, whose derivation is often unknown, since they belonged to the heritage of secret knowledge passed down from one generation of builders to the subsequent one. The design was carried out by establishing precise rules of proportion between the structural elements of the building: the equilibrium state was achieved through geometrical criteria, without any consideration of the concepts of statics, strength, safety and collapse loads [1]. This *geometrical approach* was almost abandoned during the XIX century and the favourite key to reading the behavior of the masonry arches became the elastic analysis. However, the scientific community of that time became soon aware of the ephemeral nature of the state of a masonry structure and of its close relationship with the arbitrary definition of the boundary conditions. During the second half of the XX century the plastic theory spread among the scientists and became parallel with the elastic one; the limit analysis, strongly connected to geometry, started to be considered as a valid method for the structural analysis of the masonry arches. For a detailed analysis on the historic development of the static theories on masonry arches, the reader is invited to see [2,3].

In particular, since the beginning of the studies on the stability of masonry arches through the limit analysis, carried out for the first time by Jacques Heyman during the XX century, the bearing capacity of these structures has been considered a geometric problem [4,5]. The research on the optimal shape and the minimum thickness has been a central topic for years and is still now a theme of great interest, both for the specific case of an arch [6–8] and for vaults and dome more in general [9–12]. In particular, the first solution for the minimum thickness of the circular masonry arch was provided by Milankovitch about one century ago; an interesting remark can be found in [13,14].

* Corresponding author.
E-mail addresses: nicola.cavalagli@unipg.it (N. Cavalagli), vittorio.gusella@unipg.it (V. Gusella), lauraseverini@strutture.unipg.it (L. Severini).

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Some studies have been developed regarding the effects of the stereotomy on the collapse loads multiplier. The term stereotomy – from the Greek στϵρϵόζ, solid and τομη, cut – refers to the science of the cutting of solids and uses the geometrical projections for the determination of the shape and size of the stones that form arches, vaults or domes.Differently from the main contributions that considered an arch made of voussoirs having constant thickness and joints defined by radial cuts, more recently, other authors studied the effect of a non-radial joints orientation on the minimum thickness of the arch [15], revealing that the thrust line is not unique since it depends on the shape of the voussoirs.

The effect of an irregular geometry on the bearing capacity of masonry arch bridges has been studied by de Arteaga et al. [16], through the Livesley's linear programming method combined with a detailed structural relief, performed by means of planimetric surveying techniques. The results, related to some cases study subjected to the action of a vertical pointed load, permitted to highlight that an idealized geometry may lead to an unsafe solution in term of collapse multiplier. The main influence of the thickness value on the collapse condition of the masonry arch has been highlighted by Riveiro et al. [17,18], who applied a limit analysis based procedure to an existing masonry arch bridge, whose geometry has been reconstructed in detail through a novel methodology for the three-dimensional survey [17], subsequently improved by means of an integration with non-destructive tests for the geometrical characterization of the hidden portions of the structure [18]. The influence of a local thickness reduction on the seismic capacity of masonry arches has been evaluated by Zampieri et al. [19] by means of a limit analysis procedure based on the virtual work principle. The parametric analysis on the characteristics of the defect has shown a variation of the collapse multiplier value and of the collapse mechanism, depending on the intensity and localisation of the defect itself. The effects of a localized thickness loss of the arch has been analysed also by Zanaz et al. [20], who presented a methodology for the assessment of the masonry vaults bearing capacity in presence of a pointed vertical load, based on the finite element method. All these approaches consider the irregularity of the structure by means of the regeneration of its real geometry or through the identification of a local defect, but they do not reproduce the uncertainty related to the shape of each constitutive stone element of the arch. When the analysis reveals that the structure is not able to stand the assigned loads, strengthening interventions should be realized in order to increase its bearing capacity [21–24].

The knowledge of the carrying capacity under horizontal loads is a first fundamental step toward the comprehension of the behaviour of complex structural systems in seismic areas. When evaluating the limit equilibrium condition of the masonry arch, the presence of both vertical and horizontal loads has been considered by several authors [25–30], also with the presence of the backfill for the analysis of historical bridges [31,32]. In this context, the evaluation of the ultimate resistance of the masonry arch has been usually carried out by considering deterministic values of the involved geometrical or mechanical parameters, but actually many factors can affect the deterministic ideal bearing capacity. The effects of the random variability of the material strength have been evaluated by some researchers for the circular masonry arch subjected to vertical loads, through static or kinematic approaches [33,34]. Other studies have been carried out on masonry arch bridges subjected to vertical pointed loads, in order to perform a safety assessment that takes into account the random variability of the properties of the backfill [35]. A probabilistic approach for the safety assessment of existing arches has been proposed by Schueremans et al. [36] and applied to a circular masonry arch having an uncertain geometry and subjected to a vertical pointed load. Defects on the shape of the voussoirs, due to both the imprecisions of construction and the deterioration associated to environmental actions, can often be found in masonry structures. Hence, the safety assessment of the masonry arch should take into account the variability of these factors, by adopting random values of the geometrical parameters instead of deterministic ones. To the knowledge of the authors, there are no studies about the effects of the geometrical uncertainties of the voussoirs on the collapse load of the masonry arch in presence of horizontal actions. In fact, the existing studies took into account only the presence of vertical loads or considered the real geometry of a certain case study, without reproducing the uncertain geometry of each voussoir in a probabilistic sense.

This paper deals with the uncertain geometry and its effects on the horizontal loads carrying capacity of the circular masonry arch. In the first part of this work, the limit analysis based procedure has been explained and a deterministic calculation, which refers to an ideal geometry, has been carried out in order to determine the nominal horizontal loads multiplier. Nowadays, other computational methods could be useful to carry out consistent numerical analyses, taking also into account the effective behaviour of the materials, with their hardening and softening response. Nevertheless, when a significant amount of cases has to be considered, the limit analysis is an efficient tool, which allows to achieve reliable results with low computational costs. The evaluation of the failure condition has been performed by adopting the well known Heyman's hypotheses related to the masonry, which has been considered as a no-tension material, with infinite compressive strength and capable to generate a friction between the voussoirs that prevents the sliding. In the second part, the effects of the geometrical uncertainties of the voussoirs and the imprecisions of construction have been modelled. Among the approaches that could be used to analyse the uncertainties effects on the structural response, probabilistic, fuzzy and interval methods are worthy of note [37]. The fuzzy method is generally used when the boundaries of a set of activities are not well-defined [38], while the interval method can be used when the uncertain parameters are denoted by simple ranges. However, even if the analyses performed in this work are based on the variability of some parameters within specific ranges related to geometric tolerances, the probabilistic approach has been preferred due to the great number of variables to be considered, for example in the case of high number of voussoirs. In fact, the thickness, the radius of the mean circular construction line of the arch and the angle of embrace of each voussoir have been considered as random variables with uniform probability density functions. All the other parameters involved in the problem have been taken into account as deterministic. The effects of the uncertain geometry on the horizontal loads carrying capacity have been evaluated in term of probability density function of the horizontal load multiplier.

2. Mechanical model and limit analysis

2.1. The limit analysis based method

The safety of the masonry arch has been analysed by referring to the limit analysis. A thrust line analysis has been carried out starting from the following hypotheses for the masonry: (i) no-tension material, (ii) infinite compressive strength and (iii) the sliding between the voussoirs does not occur. The applied method is based on the assumption that the arch at the collapse satisfies at the same time the equilibrium, the resistance criterion and the mechanism condition. In other words, the thrust line must be determined by imposing the equilibrium respect to the acting loads, including the self-weight, and must be contained everywhere inside the thickness of the arch in order to make the