Computers and Structures 188 (2017) 63–79

Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

Higher-order approximation to suppress the zero-energy mode in non-ordinary state-based peridynamics

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ARTICLE INFO

Article history:
Received 16 July 2016
Accepted 27 March 2017

Keywords:
Zero-energy mode
State-based peridynamics
Non-ordinary
Stability

ABSTRACT

This paper presents a numerical method to control the spurious deformation mode conventionally found in a non-ordinary state-based peridynamic formulation. The proposed approach introduces a higher-order approximation for a deformation gradient tensor in order to suppress oscillations from the zero-energy mode. The results are compared to those determined by other available methods. The findings of this study well demonstrate that the proposed method provides highly effective solutions for controlling zero-energy modes in peridynamic models. Contrasting other available zero-energy control methods, the proposed method remains robust for relatively large horizon sizes. The method is incorporated by implicit weight functions in peridynamics.

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1. Introduction

Nonlocal methods in computational solid mechanics have recently gained popularity as an alternative method to the classical continuum mechanics methods, such as the finite element method (FEM). Peridynamics introduced by Silling [1] is one of the nonlocal mesh-free methods which reformulate the classical continuum mechanics by substituting the governing partial differential equations with integral equations. Therefore, it is considered that the peridynamic equation of motion is valid everywhere despite the presence of cracks and other discontinuities [2–9].

Peridynamics has undergone significant changes since first proposed. When first introduced, so called bond-based peridynamics (BBPD) only considered independent central interaction forces between any pair of material particles. As a consequence of this assumption, the bond-based method was restricted to a fixed value of Poisson's ratio ($\nu = 1/4$ for the 3D and 2D plane strain cases and $\nu = 1/3$ for 2D plane stress cases) [4,6,8,10,11]. Due to this limitation, BBPD had lost its generality and credence for modeling material response such as plasticity. The non-ordinary state-based peridynamics (NOSBPD) is a generalized form of peridynamics which allows material particle bonds to carry forces in all directions. Therefore, the NOSBPD can represent material behavior with any Poisson's ratio [4,8,10,12–14]. Furthermore, the bond forces in the NOSBPD are defined by stress tensors; therefore, the classical constitutive equations and failure criteria can be implemented in the NOSBPD analysis framework [15–19].

However, computational problems related to the NOSBPD include an instability problem, known as a 'zero-energy mode' mechanism [17,20]. It is mainly observed in regions with high strain gradients. The term 'zero-energy mode' associated with reference to the Finite Element Analysis (FEA) refers to a nodal displacement vector that is not a rigid-body motion, yet produces zero strain energy. Instabilities arise because of weak-form element formulation processes such as a the use of a low-order Gauss quadrature rule. Certain higher-order polynomial terms vanish at Gauss points, thus eliminating these terms from contribution to the system stiffness [21,22].

In the context of peridynamics, the zero-energy mode is associated to a weak coupling of material particles with their surrounding particles and thus results in oscillations in the deformation and stress fields. Various control methods have been proposed. For example, introducing a viscosity term in the peridynamic equation of motion or decreasing particle spacing size have relieved the zero-energy mode oscillations [23]; however, another level of effort is required to significantly reduce the undesired oscillations. Several other methods have been developed by Littlewood [24], Breitenfeld et al. [17], and Wu and Ren [25] to suppress the zero-energy mode.

The zero-energy mode is also present in the other available meshfree methods such as smoothed particle hydrodynamics (SPH) and element-free Galerkin method (EFG). The meshfree methods suffer from another stability problem known as "tensile stability". Although the tensile stability is beyond the scope of this

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http://dx.doi.org/10.1016/j.compstruc.2017.03.019
0045-7949/Published by Elsevier Ltd.

study, it is worth understanding the background. The zero-energy method in meshfree methods comes from 'rank-deficiency' in evaluating the integrals of weak forms due to a relatively small number of integration points. Additional integration points were first introduced to decrease the instability of the SPH by Dyka and Ingel [26], and this method was extended for higher dimensions by Randles and Libersky [27]. In the so called "stress point integration method", additional slave points are embedded into particles created in an existing particle arrangement. The slave or stress points are only needed in evaluation of governing equations used in the weak form, and all their field values are obtained through an interpolation from the original particles. Belytschko et al. [28] illustrated that the stress point integration method does not affect the tensile instability but removes the instability due to rank-deficiency. Rabczuk et al. [29] employed a modified stress integration method to eliminate the rank-deficiency. They included the quadrature of the Galerkin weak form over both original particles and stress points. While the stress point integration method provides improved stability over nodal integration methods, it is likely to impair computational efficiency of the meshfree method, particularly in a large structural analysis problem.

Introducing a supplementary term to the force vector-state, which resembles the process of providing an artificial stiffness for stability in the classical FEM, represents an extra coupling of each particle with neighboring particles. Various forms for the supplementary term have been provided by Littlewood [24] and Breitenfeld et al. [17], However, these solutions do not provide resolutions to the fundamental problem; that is, they are mesh (or particle spacing size) sensitive. Wu and Ren [25] introduced a stabilized displacement field to control the zero-energy mode. The stabilized displacement of each particle is determined by providing a weighted average displacement of all other particles and yields reduced oscillations in the deformation field. This approach eliminated the need for the spring coefficient required for the supplementary force. However unfortunately, the oscillation problem appears to remain in the strain and stress fields with the Wu and Ren model [25].

In this paper, it is proposed to extend the first-order Taylor approximation to higher-order in order to obtain approximate deformation gradient tensor and to solve the nonlocal peridynamic equations. The proposed scheme has been implemented in 1D and 2D problems with varying horizon sizes and discretization patterns which are integral features of peridynamics. The proposed higher-order formulation is naturally included in the weight function necessary to define a deformation gradient tensor in the peridynamics framework. Three example problems are studied herein to demonstrate the capability of the proposed method for relieving the oscillations conventionally found in the displacement and stress solutions. A comparative study is performed for varying particle spacing, horizon size, and weight functions. Through the examples demonstrated in this study, it is demonstrated that the proposed method is far more effective in suppressing the zero-energy mode oscillations than existing methods described above. A nonlinear elastic-plastic simulation is performed to illustrate the capability of the proposed method as well as the considerably reduced oscillations when the method is implemented in the peridynamics analysis framework.

2. State of the research on suppressing spurious zero-energy modes in peridynamics

2.1. Non-ordinary state-based peridynamics

The kinematics of peridynamics for a 2D body is illustrated in Fig. 1. In peridynamics, a particle located at position $\mathbf{x}$, interacts

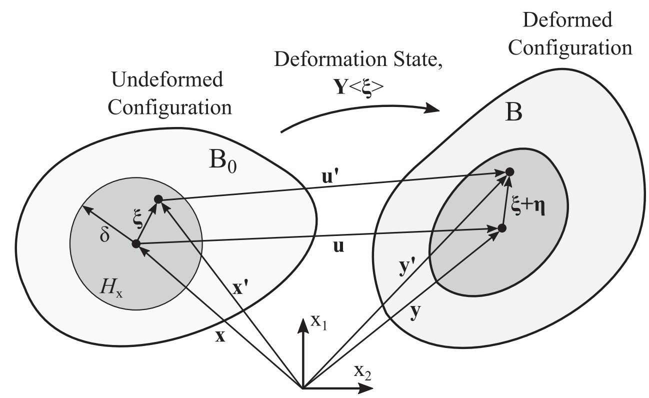

Fig. 1. A schematic peridynamic body.

with its surrounding particles within an area of influence, so called 'horizon' $\mathcal{H}_{\mathbf{x}}$, where δ is the horizon size. The position vector-state, $\mathbf{X}\langle\mathbf{x}'-\mathbf{x}\rangle=\xi=\mathbf{x}'-\mathbf{x}$, also referred to as the 'bond' between two particles $\mathbf{x}$ and $\mathbf{x}'$, represents the relative position in the undeformed body, $\mathbf{B}_0$. The deformation vector-state, $\mathbf{Y}\langle\mathbf{x}'-\mathbf{x}\rangle=\xi+\boldsymbol{\eta}=\mathbf{y}'-\mathbf{y}$, maps the bond, $\mathbf{X}\langle\xi\rangle$, in the deformed body, $\mathbf{B}$, where $\boldsymbol{\eta}=\mathbf{u}'-\mathbf{u}$ is the relative displacement of particles $\mathbf{x}$ and $\mathbf{x}'$.

In the NOSBPD, the steady-state equilibrium equations for particle $\mathbf{x}$ is given in Eq. (1) where $\mathbf{b}$ is the body force applied on the particle $\mathbf{x}$, and $dV_{\mathbf{x}'}$ is the volume of particle $\mathbf{x}'$. The force vector-state, $\mathbf{T}$, can be obtained by Eq. (2), where $\omega(\xi)$ denotes an weight function and $\xi=|\xi|=|\mathbf{x}'-\mathbf{x}|$. The stress tensor, $\boldsymbol{\sigma}$, is the first Piola-Kirchhoff stress, and the shape factor, $\mathbf{K}$, at particle $\mathbf{x}$ is defined by Eq. (3). $\otimes$ denotes the tensor product.

$$\int_{\mathcal{H}_{\mathbf{x}}}(\mathbf{T}[\mathbf{x}]\langle\mathbf{x}'-\mathbf{x}\rangle-\mathbf{T}[\mathbf{x}']\langle\mathbf{x}-\mathbf{x}'\rangle)\mathrm{d}V_{\mathbf{x}'}+\mathbf{b}(\mathbf{x})=\mathbf{0} \tag{1}$$

$$\mathbf{T}[\mathbf{x}]\langle\mathbf{x}'-\mathbf{x}\rangle=\omega(\xi)\boldsymbol{\sigma}\cdot\mathbf{K}^{-1}\cdot(\mathbf{x}'-\mathbf{x}) \tag{2}$$

$$\mathbf{K}(\mathbf{x})=\int_{\mathcal{H}_{\mathbf{x}}}\omega(\xi)[(\mathbf{x}'-\mathbf{x})\otimes(\mathbf{x}'-\mathbf{x})]\mathrm{d}V_{\mathbf{x}'} \tag{3}$$

A nonlocal approximation of deformation gradient tensor, $\mathbf{F}$, is defined by Silling et al. [4] to formulate the classical continuum mechanics' constitutive equations in peridynamics.

$$\mathbf{F}(\mathbf{x})=\left[\int_{\mathcal{H}_{\mathbf{x}}}\omega(\xi)[(\mathbf{y}'-\mathbf{y})\otimes(\mathbf{x}'-\mathbf{x})]\mathrm{d}V_{\mathbf{x}'}\right]\cdot\mathbf{K}^{-1} \tag{4}$$

The small strain tensor for isotropic elastic materials is determined by $\boldsymbol{\epsilon}=1/2(\mathbf{F}+\mathbf{F}^T)-\mathbf{I}$, where the stress is obtained by $\boldsymbol{\sigma}=\mathbf{C}:\boldsymbol{\epsilon}$, where $\mathbf{C}$ is the isotropic elastic moduli matrix; and $\mathbf{I}$ is the identity matrix. To obtain the implicit solutions for a quasi-static peridynamic equilibrium equation (Eq. (1)), an incremental and iterative method is developed. For a virtual displacement state of $\delta\mathbf{u}$, the dynamic relaxation method is adapted to iteratively update the displacement field, $\mathbf{u}$, until $||\delta\mathbf{u}||<\varepsilon$, where ε is a small numerical cut-off. Dynamic relaxation is an explicit iterative method for obtaining steady-state solutions [18,30,31]. This type of dynamic relaxation method determines steady-state solutions for a dynamic system by introducing fictitious mass and damping matrices and is particularly effective for solving highly nonlinear problems including geometric and material nonlinearities.

2.2. Available treatments for zero-energy modes

The spurious zero-energy mode oscillations in the NOSBPD solutions are attributed to weak form integration in nonlocal