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A quadratic manifold for model order reduction of nonlinear structural dynamics

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ABSTRACT

This paper describes the use of a quadratic manifold for the model order reduction of structural dynamics problems featuring geometric nonlinearities. The manifold is tangent to a subspace spanned by the most relevant vibration modes, and its curvature is provided by modal derivatives obtained by sensitivity analysis of the eigenvalue problem, or its static approximation, along the vibration modes. The construction of the quadratic manifold requires minimal computational effort once the vibration modes are known. The reduced-order model is then obtained by Galerkin projection, where the configuration-dependent tangent space of the manifold is used to project the discretized equations of motion.

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1. Introduction

The use of large Finite Element (FE) models for nonlinear structural analysis is becoming a pressing need in several industrial fields, as for instance the mechanical, aerospace and biomedical. Nowadays, it is relatively easy to generate large models that account for extremely detailed geometric features and material distribution. However, such models are often of prohibitive size and routine simulations to explore different load scenarios, geometric layouts and material choice are severely limited. Among other nonlinear effects, geometric nonlinearities mainly characterize thin-walled structural components that are typically employed when high stiffness-to-weight and strength-to-weight ratios must be achieved. The redirection of stresses due to non-infinitesimal deflections causes peculiar behaviors as bending and torsion-stretching coupling, buckling, snap-through and mode jumping [1]. In this context, Reduced Order Models (ROMs) are paramount to enable sound design and optimization activities. In a broad sense, ROMs are low order realizations of the original model, often referred as High Fidelity Model (HFM). This reduction is achieved through a projection of the full model onto a Reduced-Order Basis (ROB) which spans the subspace in which the solution is assumed to lie.

An established method to obtain accurate ROMs by Galerkin projection is the Proper Orthogonal Decomposition (POD) [2,3], where the reduction basis is constructed using the solution snapshots of the HFM. Albeit optimal in a sense, it bears the drawback of requiring the full solution. Nonetheless, it is meaningfully applied in the so-called *many-queries* scenarios, for which the cost of the full training simulations is justified. In a preliminary design context, however, the resources required for such an approach might not be available. In this case, it is desirable to build a ROM *not* with the reliance on full simulations, but rather using certain intrinsic characteristics of the underlying physical system, which are usually available at a very small fraction of the computational cost associated to such full simulation(s).

Modal truncation and superposition is a standard practice for linear structural dynamics, as it enables the decoupling of the linear governing equation to readily assess the dynamic response. However, a reduction based solely on Vibration Modes (VMs) would perform poorly in the presence of geometric nonlinearities, as they typically do not capture the relevant bending/torsion-stretching coupling. This would require the inclusion of in-plane displacement dominated fields in the basis. An appealing enrichment to a ROB of few VMs is constituted by the Modal Derivatives (MDs), which were originally proposed in [4]. These are computed by differentiating the eigenvalue problem associated to small, undamped vibrations with respect to the modal amplitudes. A static version of their construction (i.e. neglecting the inertial terms) enjoys computational advantages: the MDs thus obtained, are the solutions of a set of linear systems where the coefficient matrix is factorized only once and the right hand sides are symmetric functions of the VMs.

In a reduced-basis approach, the MDs could be appended to a ROB constituted by the dominant VMs. This approach leads to very

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accurate results [5,6]. Unfortunately, the number of MDs that can be generated, grows quadratically with the size of the VMs basis used to generate them, thereby severely hampering the efficiency of the method. However, the MDs are in fact the curvature of a quadratic manifold arising from the Taylor expansion of the physical displacement into the direction of the dominant vibration modes. As such, the modal amplitudes associated to the MDs are enslaved, in a quadratic fashion, to those of the VMs, and hence do not require independent reduced unknowns for their description. This approach can often be supported by a sound theoretical justification in examples which are characterized by a special dichotomy in time scales, and corresponds to neglecting the inertial forces associated to the fast dynamics of the system at hand [7]. More specifically, the static MDs provide a second order approximation to the underlying critical manifold in such examples [8]. This leads to the notion of the solution lying on a *quadratic* manifold, parameterized by the amplitudes of the dominant VMs. This idea already appeared in a static context when evaluating the initial post-buckling response of thin-walled structures using a perturbation approach [9,10], and, with a very similar framework, in the computation of the backbone curves for nonlinear harmonic responses [11].

The use and efficacy of such a Quadratic Manifold to construct a ROM for dynamic applications remains unexplored and is the focus of this work. In this work we propose a unified approach to construct a ROM using a quadratic manifold comprised of VMs and MDs. The classical notion of the Galerkin projection is extended here to projection on a tangent, configuration-dependent space, which is variationally consistent with the nonlinear mapping between modal and full DOFs. Further, we test this approach on a simple, illustrative example as well as a realistic, industrial structure and compare it with established reduction techniques.

It is well known that once a ROB has been constructed, significant speed-ups could be obtained by equipping the ROM with one of the many available *hyper-reduction* techniques, [12–16] which aim at scaling the cost of evaluation of the reduced nonlinear term down to the order of the number of reduced variables, and not that of the original HFM. Regardless of the specific method adopted, the accuracy of any ROM is determined by the choice of the associated reduction subpace. To this effect, this paper focuses only on the reduction subspace and its generalization to a curved manifold, and computational speed-up will not be discussed here.

This paper is organized as follows. The generalization of the Galerkin projection onto a nonlinear manifold is sketched in Section 2. The construction of a MD-based linear manifold is discussed in Section 3. The quadratic manifold is then introduced in Section 4. Numerical results are presented and discussed in 5, and finally, the conclusions are given in Section 6. The appendix describes the comparison of the proposed approach with the static condensation technique discussed in [23].

2. Model order reduction

The dynamical response of a structure to externally applied loading is obtained by solving an Initial Value Problem (IVP). This IVP is characterized by a system of second-order Ordinary Differential Equations (ODEs) usually resulting from the FE discretization of the governing Partial Differential Equations (PDEs), and can be written in the following form:

$$\begin{aligned}&\mathbf{M}\ddot{\mathbf{u}}(t)+\mathbf{C}\dot{\mathbf{u}}(t)+\mathbf{f}(\mathbf{u}(t))=\mathbf{g}(t)\\&\mathbf{u}(t_0)=\mathbf{u}_0\\&\dot{\mathbf{u}}(t_0)=\mathbf{v}_0,\end{aligned} \tag{1}$$

where the solution $\mathbf{u}(t)\in\mathbb{R}^n$ is a high dimensional generalized displacement vector with the initial conditions $\mathbf{u}_0$ for displacements and $\mathbf{v}_0$ for velocities given as inputs at initial time $t_0, \mathbf{M}\in\mathbb{R}^{n\times n}$ is the mass matrix, $\mathbf{C}\in\mathbb{R}^{n\times n}$ is the damping matrix, $\mathbf{f}(\mathbf{u}):\mathbb{R}^n\mapsto\mathbb{R}^n$ is the nonlinear internal force and $\mathbf{g}(t)\in\mathbb{R}^n$ is the time dependent external load vector. These ODEs are further discretized in time using a suitable time integration scheme, resulting in a high-dimensional, fully discrete, nonlinear system of algebraic equations, to be iteratively solved at each time step with a Newton method (for example). This full solution bears a prohibitive computational cost even for a single-query scenario, not to mention the case when the time integration needs to be performed several times, e.g., to explore different operational scenarios.

Fortunately, in structural dynamics applications, a relatively small number of "modal" coordinates are expected to govern the system response. This is to say that, in general, the solution may be assumed to evolve on a low dimensional manifold in $\mathbb{R}^n$. In other words, we seek a mapping $\boldsymbol{\Gamma}:\mathbb{R}^n\mapsto\mathbb{R}^m$ with $m\ll n$ such that

$$\mathbf{u}(t)\approx\boldsymbol{\Gamma}(\mathbf{q}(t)), \tag{2}$$

where $\boldsymbol{\Gamma}$ is a general nonlinear mapping and $\mathbf{q}\in\mathbb{R}^m$ is the reduced vector of unknowns. The semi-discrete equations for dynamic equilibrium in (1) can be written in the following variational or weak form (time dependency is omitted for clarity purposes):

$$[\mathbf{M}\ddot{\mathbf{u}}+\mathbf{C}\dot{\mathbf{u}}+\mathbf{f}(\mathbf{u})]\cdot\delta\mathbf{u}=\mathbf{g}\cdot\delta\mathbf{u}, \tag{3}$$

where $\delta\mathbf{u}$ is an admissible variation in the solution $\mathbf{u}$. By introducing the lower dimensional approximation (2) into (3), we obtain

$$\left[\mathbf{M}\ddot{\boldsymbol{\Gamma}}(\mathbf{q})+\mathbf{C}\dot{\boldsymbol{\Gamma}}(\mathbf{q})+\mathbf{f}(\boldsymbol{\Gamma}(\mathbf{q}))\right]\cdot\delta\boldsymbol{\Gamma}(\mathbf{q})=\mathbf{g}\cdot\delta\boldsymbol{\Gamma}(\mathbf{q}). \tag{4}$$

Observing that the variation $\delta\boldsymbol{\Gamma}(\mathbf{q})$ is given by $\frac{\partial\boldsymbol{\Gamma}(\mathbf{q})}{\partial\mathbf{q}}\delta\mathbf{q}$, and $\delta\mathbf{q}$ being arbitrary, we finally obtain

$$\mathbf{P}_{\Gamma}^T\left[\mathbf{M}\ddot{\boldsymbol{\Gamma}}(\mathbf{q})+\mathbf{C}\dot{\boldsymbol{\Gamma}}(\mathbf{q})+\mathbf{f}(\boldsymbol{\Gamma}(\mathbf{q}))\right]=\mathbf{P}_{\Gamma}^T\mathbf{g}, \tag{5}$$

where $\mathbf{P}_{\Gamma}$ denotes the tangent subspace $\frac{\partial\boldsymbol{\Gamma}(\mathbf{q})}{\partial\mathbf{q}}$.

If the mapping function is chosen to be linear such that $\boldsymbol{\Gamma}(\mathbf{q}):=\mathbf{V}\mathbf{q}$ (where $\mathbf{V}\in\mathbb{R}^{n\times m}$ is typically a basis spanning some lower dimensional subspace in $\mathbb{R}^n$ in which the solutions is assumed to live), the above treatment leads to the *Bubnov-Galerkin* or simply the *Galerkin Projection*. The reduced ODEs can then be simplified as

$$\underbrace{\mathbf{V}^T\mathbf{M}\mathbf{V}}_{\tilde{\mathbf{M}}}\ddot{\mathbf{q}}(t)+\underbrace{\mathbf{V}^T\mathbf{C}\mathbf{V}}_{\tilde{\mathbf{C}}}\dot{\mathbf{q}}(t)+\underbrace{\mathbf{V}^T\mathbf{f}(\mathbf{V}\mathbf{q}(t))}_{\tilde{\mathbf{f}}(\mathbf{q}(t))}=\mathbf{V}^T\mathbf{g}(t), \tag{6}$$

where $\tilde{\mathbf{M}},\tilde{\mathbf{C}}\in\mathbb{R}^{m\times m}$ are the reduced mass and damping matrices, respectively. For a linear system, one would have $\mathbf{f}(\mathbf{u})=\mathbf{K}\mathbf{u}$ ($\mathbf{K}\in\mathbb{R}^{n\times n}$ being the linear stiffness matrix), and a reduced stiffness matrix $\tilde{\mathbf{K}}=\mathbf{V}^T\mathbf{K}\mathbf{V}\in\mathbb{R}^{m\times m}$ is also obtained.

The choice of projection basis $\mathbf{V}$ (or the mapping $\boldsymbol{\Gamma}(\mathbf{q})$) is critical in determining the accuracy of the reduced solution. The size of the basis (or the reduced number of unknowns) is important in determining the speed-up in computation time. In further sections, we consider the candidates for such linear and nonlinear mappings.

3. Linear manifold

The existence of an invariant subspace (or manifold) is a key requirement for reduction of system (1), as described above. Upon reduction over an invariant *linear* subspace, we refer to the reduced solution to lie on a *Linear Manifold*. Finding a suitable invariant subspace is by no means trivial, if at all possible. In the linear mappings context, the POD is a remarkably versatile and robust method. However, one of its drawbacks is the need for training snapshots of solution vectors which are obtained from a full nonlinear run. Typically, a reduction basis constructed in such a manner is suitable

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