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Material softening and strain localization in spatial geometrically exact beam finite element method with embedded discontinuity

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ABSTRACT

When some critical condition is reached at a material point of a solid body, a localized strain starts developing which makes the strain field discontinuous and highly accelerates local damaging of material. The present paper addresses this kind of strain localization in spatial geometrically exact beams. Here we propose a new beam finite element formulation which accounts for softening of material by applying the embedded strong strain discontinuity technology. The formulation is essentially an extension of the original strain-based formulation, upgraded such to allow for detecting the onset of strain localization and to introduce additional equations for evaluating singular strain peaks and jumps of displacements and rotations at the localized section in further deformation. The consistency condition that the equilibrium and the constitutive stress-resultants are equal is shown to be naturally suited for the implementation into the discontinuous formulation. The condition for the onset of strain localization at a beam cross-section is here related to the loss of uniqueness of the beam cross-sectional constitutive equations. If the condition for a unique inverse is violated, two solutions are possible for cross-sectional strains. In a subsequent deformation, one of the two solutions follows the softening regime of material. The discontinuous increments in strains, displacements and rotations at the softening cross-section are obtained from the equations of the structure supplemented by the consistency conditions of the softening cross-section.

1. Introduction

Softening often takes place in damaged heterogeneous materials at some deformation. When an appropriate material and stress dependent critical condition is reached at some material point of a solid body during the subsequent deformation, a thin band of a localized strain occurs in the softened material which further accelerates local damaging of material. This physical phenomenon, typically marked as the strain localization, can well be mathematically modelled, if the band is assumed to be a surface of zero width and the strains taken to be peak-like discontinuous across the surface. Such a singular discontinuity of strains occurs due to a local loss of uniqueness of the solution. The proof that the strain softening rate-independent plasticity material implicates singular distributions in the strain field was discussed in Simo et al. [1].

This interesting, although a rather complicated theoretical problem of a large practical value has motivated a research community for long, in looking for stable, reliable and computationally efficient numerical solution methods for solid bodies as well as for beams, plates and shells.

Probably the simplest solution method first introduced by Bažant et al. [2] employed a band finite element, specially designed to simulate softening in the critical point. The method is successful in the regularization of the singularity problem, but requires an internal length scale in the constitutive model and is impractical if the location of the critical point cannot be estimated in advance. See, e.g. [3,4] for such a numerical formulation of a planar geometrically exact reinforced concrete beam based on a high-order strain interpolation in regular beam elements and on a constant-strain band element in the localized zone, and Češarek et al. [5] for its extension to space. This kind of finite element formulation is considered to be a member of a broad class of non-local continuum models where the material response at a point is assumed to depend on deformations of its finite neighbourhood [6–9]. Further regularization techniques have been proposed, amongst which the higher-gradient models introduce additional higher order terms in the constitutive laws [10]; the Cosserat continuum techniques involve the local rotations as independent kinematic variables [11]; the smeared crack models where the energy dissipation associated to strain softening is distributed over the volume of finite

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elements [12]. Each of these methods has its own advantages, but the internal scale parameter of a constitutive model still seems to remain a major problem. Another technique is the cohesive zone model proposed by Needleman [13,14] where the discontinuities propagate only along the element boundaries using special cohesive elements. This model requires adaptive re-meshing techniques. For a recent and a more detailed review of various techniques, we refer to [15].

It appears that the embedded discontinuity approach has dominated recently, see, e.g. Refs. [1,16–30], among many others. The main idea of the embedded strong discontinuity approach is to enhance the standard continuous strain field within a finite element with a point-wise, peak-like discontinuous strain increment. Such finite elements are then able to model the discontinuous nature of the localized strain and displacement fields at the element level, preserving, simultaneously, the chosen degree of standard interpolations for the continuous part of the strains or displacements. After the discontinuous strain and displacement increments are eliminated from the governing equations of a finite element, the resulting element can be employed in finite element programmes in a standard way.

In the present paper we propose a new spatial geometrically exact beam finite element formulation accounting for softening of material, and apply the embedded strong discontinuity technology. The formulation is essentially an extension of the original strain-based formulation of Zupan and Saje [31] to include softening of material with emerging peak-like discontinuous strain fields and their subsequent localization. The original formulation is upgraded to allow for detecting the onset of strain localization, i.e. finding the critical load level and spotting the critical cross-section, and to introduce additional equations to obtain the strain jumps at the localized section. Further details of the finite element implementation and the solution procedure are given later on in the text. We emphasize that the present formulation uses the stress–strain relations of material fibre to compute current values of stress-resultants as functions of strains; hence, the numerical integrations of stresses over the cross-sections at integrations points of the beam axis are required in each iteration step.

Finally, in order to demonstrate the computational behaviour of the formulation of spatial geometrically exact beam-like frames in a softening regime, we present comprehensive analyses of three representative cases. Two of them deal with softening behaviour of reinforced-concrete structures. Various results including stress distributions over the localized cross-sections and the graphs of the external force as a function of nodal displacements are displayed to show the results and the range of possible applications of the present formulation.

2. Formulation of the spatial geometrically exact beam model with discontinuous kinematics

2.1. Displacements, rotations and strains

The geometrically exact finite-strain beam theory reduces the strain tensor field, continuously distributed over the beam volume, to two strain vector fields associated with material points of the beam axis [32]. These are the translational strain vector, γ_G, and the rotational strain vector, $\boldsymbol{\kappa}_G$. Their components, $\gamma_1,\ \gamma_2,\ \gamma_3$ and $\kappa_1,\ \kappa_2,\ \kappa_3$ in the current ortho-normal body basis, G, of the rotated cross-section represent, respectively, the axial and shear strains, and the torsional and bending strains. The position of a material point on the beam axis relative to the referential point on the axis is described by the undeformed arc-length coordinate, here denoted as x. From the virtual work principle it is derived that the strain-related displacement vector, $\boldsymbol{r}_g$, of the beam axis, and the rotational vector of the cross-section, $\boldsymbol{\vartheta}_g$, as well as their variations, $\delta\boldsymbol{r}_g$ and $\delta\boldsymbol{\vartheta}_g$, all being functions of x, satisfy the following conditions:

$$\delta\boldsymbol{r}'_g(x) = \mathbf{R}(x)\delta\gamma_G(x) - \boldsymbol{r}'_g(x) \times \delta\boldsymbol{\vartheta}_g(x), \tag{1}$$

$$\delta\boldsymbol{\vartheta}'_g(x) = \mathbf{R}(x)\delta\boldsymbol{\kappa}_G(x). \tag{2}$$

Here lower indices G and g mark, respectively, the vector basis to which the particular vector components belong. g is a fixed-in-space Cartesian ortho-normal basis with coordinates X, Y, Z. In contrast the body basis, G, is fixed to the centroid of the cross-section such that its current rotated position in space agrees with the current position of the cross-section. Furthermore, as cross-sections along the beam axis experience different rotations, the body basis varies with x and thus generally differs at each cross-section. Body base vectors are chosen such that base vector $\boldsymbol{G}_1$ is perpendicular to the rotated cross-section, and base vectors $\boldsymbol{G}_2$ and $\boldsymbol{G}_3$ coincide with the directions of the principal inertia axes of the cross-section. $\mathbf{R}$ is the rotation matrix. It rotates the spatial (fixed) basis, g, onto the deformed body basis G. The rotation matrix is parametrized with the rotational vector $\boldsymbol{\vartheta}_g$. The prime ($'$) marks the derivative with respect to x.

Integrating Eqs. (1) and (2) with respect to the variations gives the relation between displacement, rotation and strain vectors:

$$\boldsymbol{r}'_g(x) = \mathbf{R}(x)\big(\gamma_G(x) - \gamma^0_G(x)\big), \tag{3}$$

$$\boldsymbol{\vartheta}'_g(x) = \mathbf{T}^{-T}(x)\big(\boldsymbol{\kappa}_G(x) - \boldsymbol{\kappa}^0_G(x)\big). \tag{4}$$

The details of the derivation are presented in, e.g. [31,33,34]. In Eq. (4) $\mathbf{T}$ denotes the transformation matrix operating between $\boldsymbol{\kappa}_G - \boldsymbol{\kappa}^0_G$ and $\boldsymbol{\vartheta}'_g$ [31,33,34]. Functions $\gamma^0_G(x)$ and $\boldsymbol{\kappa}^0_G(x)$ are arbitrary initial strain vectors of the unloaded beam; they do not change during deformation and are thus variational constants. In the numerical examples that follow, we will assume that the beam is straight initially so that $\gamma^0_G = [-1, 0, 0]$ and $\boldsymbol{\kappa}^0_G = [0, 0, 0]$.

2.2. Internal forces. Constitutive equations of the cross-section and uniqueness of their inverse.

In beam theories the stress tensor field, distributed over the beam volume, is replaced by the cross-sectional stress resultant vectors, here denoted in the matrix form by $\boldsymbol{N}^C_G$ and $\boldsymbol{M}^C_G$, and called, respectively, the internal force and moment vectors, all having their points of application at the beam axis. Their components, $N^C_1,\ N^C_2,\ N^C_3$ and $M^C_1,\ M^C_2,\ M^C_3$, represent, respectively, the stress-based axial and shear forces, and torsional and bending moments, all with respect to the current rotated basis G of the cross-section. The stress resultants are the work-complements to strain vectors γ_G and $\boldsymbol{\kappa}_G$, introduced in the previous section. As the components of the stress tensor depend on the components of the strain tensor through the constitutive equations of material, and as both tensors vary over the cross-section in some given way, the cross-sectional stress resultant vectors can be shown to depend directly on the beam strain vectors. Because they describe the cross-sectional behaviour rather than solely that of material, the relations between the stress-resultant and strain vectors are called the 'constitutive equations of the cross-section'. They are functions of both the stress–strain law of material and the shape of the beam cross-section, and are here assumed in a rather general algebraic form:

$$\boldsymbol{N}^C_G(x) = \mathcal{C}_N(\gamma_G(x), \boldsymbol{\kappa}_G(x)), \tag{5}$$

$$\boldsymbol{M}^C_G(x) = \mathcal{C}_M(\gamma_G(x), \boldsymbol{\kappa}_G(x)), \tag{6}$$

where the components C^N_i and C^M_i $(i = 1, 2, 3)$ of vector functions $\mathcal{C}_N = [C^N_1,\ C^N_2,\ C^N_3]^T$ and $\mathcal{C}_M = [C^M_1,\ C^M_2,\ C^M_3]^T$ are assumed to be at