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A meshfree unit-cell method for effective planar analysis of cellular beams

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ABSTRACT

This paper presents a novel approach for accurate and efficient planar response analysis of cellular beams, which provides the necessary input for local out-of-plane buckling analysis of web components. The proposed approach utilises the super-element concept defined for unit-cells, achieving further efficiencies through an enhanced Element-Free Galerkin (EFG) approach for establishing the planar super-element response. Several examples are presented, firstly at the level of unit-cells, where the computational benefits of the EFG method are highlighted, and finally at the overall level of cellular beams, where the superior performance of the unit-cell approach with virtually no compromise in accuracy is demonstrated.

1. Introduction

Cellular beams have become in recent years a popular form of steel construction, largely due to enabling large spans while facilitating the integration of services within the floor depth [1–3]. As illustrated in Fig. 1, cellular beams are easily fabricated from standard I-beams through appropriate cutting, offsetting and joining operations, enhancing the bending stiffness and resistance via an increased distance between the flanges, while reducing weight via the resulting openings in the web.

Besides the above relative structural benefits of cellular beams, their structural response is very different in certain respects from the parent I-beam [4]. The existence of regular holes influences the structural behaviour particularly in the web region where local elements, such as the web-posts and top/bottom tees, are subject to local actions throughout the beam span. These actions consist of not only contributions to the overall cross-sectional shear and bending moment, but are also superimposed by secondary effects leading to a highly nonlinear stress distribution over the cellular beam depth and length [5,6]. One of the most important and distinctive structural characteristics of cellular beams relates to local buckling of the web elements [6–9], which in turn depends on the nonlinear distribution of planar stresses in the perforated web.

Although some guidance on the design and assessment of cellular beams is provided in codes of practice [10,11], the selection and design of cellular beams is typically undertaken in practice with the aid of specialist software (e.g. [12]). However, in considering the local buckling of web elements, simplified models were adopted [13–17], utilising for example a strut buckling analogy with empirical calibration against nonlinear finite element models [18–20]. While the resulting simplified models are computationally efficient, these are not rational and sufficiently general for application to a wide range of cellular beam geometric configurations, as they ignore both the nonlinear planar stress distribution and the two-dimensional plate buckling behaviour of the web components. Of course, accurate predictions can be achieved in this respect using nonlinear finite element analysis, though the computational and modelling demands of such an approach are still prohibitive for application in design practice.

The present work is motivated by the need for an efficient and practical modelling approach for predicting the nonlinear planar stress distribution in cellular beams, which could in turn be used for local buckling assessment of the web components in a separate model of the nonlinear out-of-plane response. Focusing on elastic local buckling as a performance consideration that may be used along with material strength limit for cellular beam design, this paper is mainly concerned with the accurate and efficient determination of the nonlinear planar stress distribution in cellular beams arising under planar loads within the linear elastic range. The

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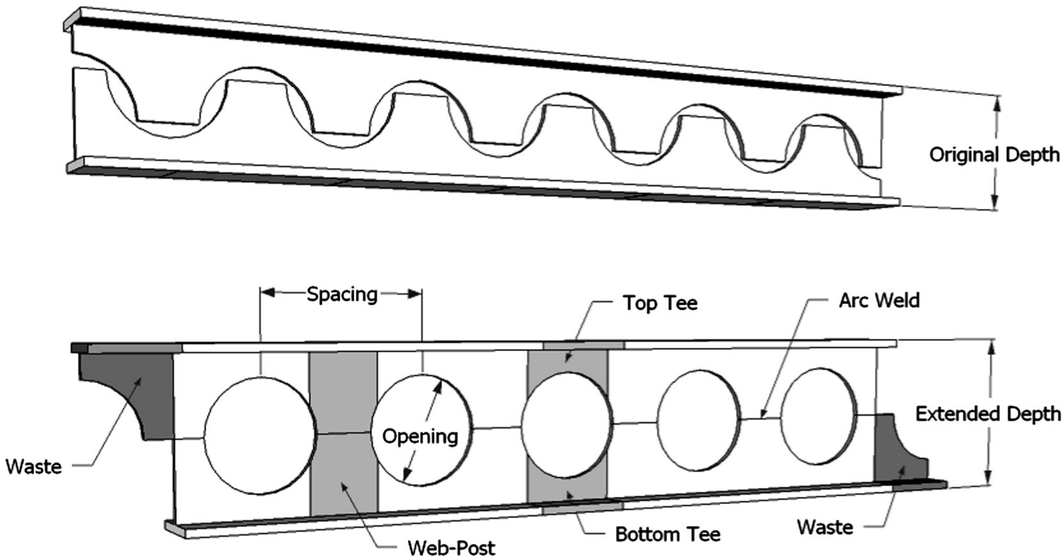

Fig. 1. Typical profiling of cellular beams.

assessment of local buckling could then be undertaken considering the out-of-plane geometric stiffness associated with the predicted planar stress distribution as well as the out-of-plane material stiffness, though this aspect is not covered in the present paper but is referred to previous work by the authors on beams with irregular openings [21].

For an accurate prediction of the nonlinear planar stress distribution in linear elastic cellular beams, the most obvious approach is to utilise finite element analysis, though this can still be computationally demanding for typical cellular beams, even in the linear elastic range. Meshfree methods have recently emerged as an alternative approach [22–24] with potentially significant relative benefits, particularly where geometric features complicate the meshing with finite elements, as in the case of beams with openings. Moreover, in such cases, the meshfree approach has the potential to achieve significant computational savings with a considerable reduction in the number of degrees of freedom (DOF) for comparable accuracy to finite element analysis.

Amongst the various meshfree methods, the Element-Free Galerkin (EFG) method [25] and the meshless local Petrov-Galerkin (MLPG) method [26] have been considered for the present work. In contrast with the MLPG method which is claimed as 'truly meshless', the EFG method requires 'background cells' for numerical integration of the governing equation over the problem domain. While this may be seen as a shortcoming for the EFG method, it is actually a benefit in the present context, since the evaluation of planar stresses at a fixed set of integration points, rather than a variable set associated with the testing function as in the MLPG method, is required for determining the out-of-plane geometric stiffness in subsequent local buckling assessment. Furthermore, the MLPG method suffers from the fact that rigid body testing modes are not accurately represented, leading to only an approximate satisfaction of equilibrium between the applied loading and the reactions at the support boundaries. Accordingly, the EFG method is selected with due consideration of its following benefits: (i) it is a meshless method that can be easily applied to complex geometric domains, (ii) it facilitates numerical integration via the use of fixed integration points over the domain, and (iii) it ensures external equilibrium at sub-domain level between loading and boundary actions.

Besides the selection of the EFG approach, this work capitalises on a major characteristic of cellular beams, which is the repetitive nature of the holes over the beam length, thus introducing the notion of a unit-cell as a 'super-element' which interacts with adjacent unit-cells via a reduced number of DOF, as illustrated in Fig. 2. Towards this end, the EFG method is adopted and suitably enhanced for planar analysis at the unit-cell level, while the overall response of the cellular beam is efficiently obtained via the assembly of super-element contributions.

The paper proceeds with presenting the numerical discretisation of a unit-cell using the EFG method, highlighting the enhancements undertaken for application to cellular beams. The formulation of the super-element associated with the unit-cell is then provided, including the treatment of edge unit-cells and the assembly of the overall cellular beam response. Finally, several examples are presented, ranging from the level of an individual unit-cell to a whole cellular beam, which demonstrate the accuracy and computational benefits of the proposed approach towards predicting the planar response of cellular beams.

2. Numerical discretisation with EFG method

The Element-Free Galerkin (EFG) method is used to discretise the problem domain at the unit cell level. The main feature of the EFG method is the utilisation of a smooth and continuous domain function via the application of a moving least squares (MLS) approach. As this MLS approximation requires only a set of nodes to generate shape functions, it is acknowledged as an element-free method which offers significant savings on computational effort, since a major cost of element meshing is eliminated [25]. However, such benefit comes with some additional costs, especially in relation to the imposition of boundary conditions, since the MLS function does not represent the actual nodal values.

The EFG method is constructed on the basis of a Galerkin formulation with the adoption of moving least-squares approximation to produce the shape functions, as presented in detail by [25]. To view the application of EFG method in 2D solid mechanic problems, a domain of Ω with a boundary Γ, being in a state of equilibrium, is considered:

$$\boldsymbol{L}_d^T\boldsymbol{\sigma} + \boldsymbol{b} = 0 \quad \text{in } \Omega \tag{1}$$

with boundary conditions specified as: