Computers and Structures 182 (2017) 392–403

Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

Response sensitivity analysis for plastic plane problems based on direct differentiation method

Yong Li [a], Surong Huang [b,*], Chun Lin [c], Quan Gu [c], Zhijian Qiu [d]

[a] Department of Civil and Environmental Engineering, University of Alberta, Edmonton T6G 1H9, Canada
[b] School of Aerospace Engineering, Xiamen University, Xiamen 361005, China
[c] School of Architecture and Civil Engineering, Xiamen University, 361005, China
[d] Department of Structural Engineering, University of California, San Diego, La Jolla, CA 92037, USA

ARTICLE INFO

Article history:
Received 2 October 2016
20 November 2016
Accepted 21 December 2016
Available online 9 January 2017

Keywords:
Response sensitivity
Direct differentiation method
Plane stress
Plane strain
Plasticity-based constitutive models

ABSTRACT

Response sensitivity analysis is of significant value to solve various inverse problems in engineering practice using gradient-based optimization algorithms. In the context of finite element (FE) method, an efficient, accurate and general sensitivity analysis approach, namely the direct differentiation method (DDM), has been developed and extended to various element and material models. However, the DDM has not been addressed for the response sensitivity of plastic plane problems (i.e., plane stress and plane strain problems), in spite of their wide applications in practice. This paper bridges this gap by extending the DDM based response sensitivity algorithm to general plastic plane problems, which is solved by taking advantage of general three dimensional (3D) constitutive models. The newly developed DDM-based sensitivity analysis algorithm is implemented in the open source finite element framework (*OpenSees*) and verified using two realistic application examples of plane problems, i.e., a concrete dam and a steel shear wall. The efficiency and accuracy of the DDM are verified by using the forward finite difference method.

1. Introduction

Parallel with the development and applications of finite element (FE) response analysis of structural and/or geotechnical systems, the FE response sensitivity analysis (RSA) has been developed for its significant value in solving various optimization problems in engineering practice using gradient-based optimization algorithms. The FE RSA has been widely used in hotspot research areas to tackle advanced engineering problems. These areas include the simplified probabilistic response analysis for performance assessment [1–3], finite element model updating [4,5], system identification [6,7], health monitoring [8], structural reliability [9], structural control, and structural optimization [10]. Recognizing the practical value of RSA for these complicated engineering problems, several methods have been developed for RSA, e.g., the finite difference method (FDM), the perturbation method (PM), the adjoint method (AM), and the direct differentiation method (DDM). DDM, compared to other existing methods with different limitations, is efficient (i.e., computationally less expensive), accurate (i.e., non-sensitive to numerical noise) and general for various nonlinear problems (i.e., not limited to linear problems) [11–15]. In each time or load step, the DDM differentiating analytically the time- and space-discretized equations of motion with respect to the sensitivity parameters, and obtaining 'exact' first derivatives of the responses of the discretized system. Therefore the DDM exhibits great efficiency and accuracy and can be extended to general nonlinear analysis by using finite element method. The authors and other researchers [11,16] have significantly contributed to the development of DDM by extending the current FE-based response analysis framework to RSA of a wide range of problems. The DDM has been extended to nonlinear structural models with multi-point constraints [12], and to various element, section and nonlinear material models [17]. Furthermore, the DDM has been extended to RSA of geotechnical systems [15,18], soil-structure interaction problems [19] and dam-reservoir-foundation systems [20]. The RSA framework has been enhanced by developing the DDM-based sensitivity analysis for various materials models (e.g., uniaxial material for concrete or steel fibers, three-dimensional (3D) constitutive models for concrete or soils), elements (e.g., displacement-based and force-based beam-column elements, quad elements, etc.), and analysis algorithms (e.g., multiple constraint handlers) in a general open source FE analysis framework, *OpenSees* [12,21–23]. To the end,

* Corresponding author.
E-mail address: srhuang@xmu.edu.cn (S. Huang).

http://dx.doi.org/10.1016/j.compstruc.2016.12.010
0045-7949/

DDM has been extended to a wide range of elements and materials for frame-type models and 3D continuum-based models (e.g., 3D building/bridge structural foundation systems) [20,24–25]. However, the FE RSA of plane problems (i.e., plane stress and plane strain problems) using DDM has not been addressed, in spite of their wide applications in engineering practice.

The 3D FE analyses are commonly used for response prediction and performance assessment to tackle real engineering problems of practical interest. However, 3D FE modeling is not necessary for the problems that can be simplified into two dimensional (2D) plane problems due to the inherent mechanical behavior. These 2D problems can be satisfactorily solved using plane analysis in an efficient way, thus significantly reducing the computational effort [26]. In practice, there are two types of fundamental plane problems (i.e., plane stress and plane strain problems) that can be formulated, assuming that certain conditions on the stress and displacement fields are satisfied, respectively [27]. For example, a plane stress problem can be defined, when the geometry of the body is such that one dimension (i.e., thickness) is usually much smaller than the other two dimensions and the external loads are uniformly applied over the thickness (e.g., thin plates, membranes, composite laminates) [28]. The stresses associated with the thickness direction are ignored in plane stress problems. On the other hand, a plane strain problem can be defined, when the geometry of the body is such that one dimension (i.e., length) is much larger than the other two dimensions (e.g., dams and tunnels) and the external loads are uniformly distributed and perpendicular to this dimension. The strains associated with the length direction are ignored in plane stress problems. The plane stress and plane strain problems are two types of fundamental mechanical problems in the structural response analysis based on continuum mechanics and are frequently encountered in engineering practice (e.g., foundations, dams, plates, composites, etc.).

In the past few years, several existing plane analysis methods have been well documented in the literature [29–31]. It is relatively straightforward to solve plane problems with homogeneous and linear elastic isotropic materials by deducing the constitutive relation between the 2D strain and 2D stress components based on the Hooke's law [32]. However, in real engineering problems, the materials are far more complicated than linear elastic isotropic and the complexity of plane problems is increased significantly when using advanced plasticity-based constitutive models. There are mainly two classes of constitutive models for the 2D plane problems, namely the direct approaches and the indirect approaches. In the direct approach, 2D constitutive models relating the stress and strain states are developed directly to simulate the responses of plane problems [33]. This approach is straightforward but usually requires extra efforts on developing new 2D constitutive models for plane problems. Contrast to the direct approach, the indirect approach takes advantage of the existing general 3D constitutive models and tailors them to 2D plane problems by providing 2D material wrappers. The 2D material wrapper allows the computation of 2D material responses from any 3D material constitutive models [32]. The indirect approach has been attractive to engineers and widely used in addressing plane problems when the 3D constitutive models are available.

This paper aims at enhancing the RSA capabilities of FE models for plane problems. Inspired by the indirect approach to address plane problems in response analysis, this paper presents a novel RSA method for plane problems by extending the DDM framework to the indirect approach, taking advantaging of the existing DDM algorithm for the 3D material models. The newly proposed algorithms are implemented in *OpenSees*. For verification purposes, two realistic application examples of plane problems, including a plane stress problem (i.e., a steel shear wall simulated using simplified J2 elastoplastic model) and a plane stress/strain problem (i.e., a concrete gravity dam simulated using truncated Drucker-Prager model), are presented in this paper. The nonlinear responses and response sensitivities are computed using the newly developed algorithms. The forward finite difference (FFD) method is used to validate the DDM formulation and to explore the benefits of DDM. The methods presented in this paper bridges an important gap between FE response-only analysis and DDM-based FE RSA for plane problems, with a wide range of engineering applications. The DDM-based RSA for plane stress and plane strain problems will contribute to the further success of other advanced problems mentioned earlier in engineering and research communities by the enhanced capability of gradient computation.

2. Response sensitivity analysis (RSA)

The responses of structural and/or geotechnical systems can be deemed as implicit mathematical nonlinear functions of the modeling parameters (e.g., material, geometric, loading, boundary parameters), which can be computed by the FE response analysis (RA). In contrast, RSA computes the first-order partial derivative of the structural responses with respect to various modeling parameters defining a structure and its loading environment. Undoubtedly, RSA requires more computational effort, but equally important, if not more than RA. RSA is applicable for various subfields of structural and geotechnical engineering using gradient-based optimization methods, such as FE model updating, system identification, health monitoring, reliability analysis, structural control, and structural optimization. Therefore, DDM has increasingly attracted more research interest due to its advantages in computing response sensitivity and the wide applications of response sensitivities to solve complicated engineering problems.

3. Direct differentiation method (DDM)

The DDM calculates the first-order derivative of the responses by differentiating analytically the governing response equations of motion after FE discretization, which involves computing the derivatives at different levels (i.e., structure, element, section, and material levels) of FE response quantities. Without loss of generality, the formulation of DDM for dynamic loading problems is presented herein briefly. In the context of FE response analysis, after spatial discretization, the partial differential equation governing the motion of the structural system takes the form,

$$\mathbf{M}(\theta)\ddot{\mathbf{u}}(t,\theta) + \mathbf{C}(\theta)\dot{\mathbf{u}}(t,\theta) + \mathbf{R}(\mathbf{u}(t,\theta),\theta) = \mathbf{F}(t,\theta) \tag{1}$$

where $\mathbf{M}$ is the mass matrix, $\mathbf{C}$ is the damping matrix, $\mathbf{R}(\mathbf{u}(t,\theta),\theta)$ is the inelastic restoring force, $\mathbf{F}(t,\theta)$ is the dynamic load applied, $\mathbf{u}(t,\theta)$ is the nodal displacement vector, t is time and the superposed dot operator represents the differentiation with respect to time, and θ is the vector of model parameters for sensitivity analysis. The dynamic equation can be further discretized along time space using numerical integration method, e.g., Newmark – β method [34], i.e.,

$$\ddot{\mathbf{u}}_{n+1} = \left(1 - \frac{1}{2\beta}\right)\ddot{\mathbf{u}}_n - \frac{1}{\beta(\Delta t)}\dot{\mathbf{u}}_n + \frac{1}{\beta(\Delta t)^2}(\mathbf{u}_{n+1} - \mathbf{u}_n) \tag{2}$$

$$\dot{\mathbf{u}}_{n+1} = (\Delta t)\left(1 - \frac{\alpha}{2\beta}\right)\ddot{\mathbf{u}}_n + \left(1 - \frac{\alpha}{\beta}\right)\dot{\mathbf{u}}_n + \frac{\alpha}{\beta(\Delta t)}(\mathbf{u}_{n+1} - \mathbf{u}_n) \tag{3}$$

Note that the dependence on parameter is omitted for simplicity in notation. Substituting Eqs. (2) and (3) into Eq. (1) yields the nonlinear equation of motion with the unknowns $\mathbf{u}_{n+1} = \mathbf{u}(t_{n+1})$,

$$\psi(\mathbf{u}_{n+1}) = \tilde{\mathbf{F}}_{n+1} - \left[\frac{1}{\beta(\Delta t)^2}\mathbf{M}\mathbf{u}_{n+1} + \frac{\alpha}{\beta(\Delta t)}\mathbf{C}\mathbf{u}_{n+1} + \mathbf{R}(\mathbf{u}_{n+1})\right] = \mathbf{0} \tag{4}$$