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An iterative algebraic dynamic condensation method and its performance

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A B S T R A C T

A novel iterative reduced-order modeling method is proposed, which is based on the recently developed algebraic dynamic condensation method. The algebraic substructuring technique is employed to improve the reduction efficiency, and the initial reduced model is calculated using the substructural stiffness condensation and the interface boundary reduction procedures. Then, the initial reduced model is iteratively updated using the iterative substructural inertial effect condensation procedure until the solutions converge. The iterative formulation of the reduced model is represented simply with small submatrix operations to avoid huge computational cost induced by the iterative procedure resulting from the very large global transformation matrix. To verify the performance of the proposed method, we consider several large structural problems, and compare the numerical results to those of the iterated improved reduced system (IIRS) method, a widely used reduced-order modeling method.

1. Introduction

While computational resources have been greatly improved, the demand for dynamic analysis of large and complex structural systems, which are modeled using the finite element (FE) method, has increased even more rapidly. Because dynamic analysis using global size matrices can be very time-consuming work, dynamic condensation methods [1–11] have been widely used for several decades. The reduced-order models obtained via dynamic condensation methods are very important in a number of research fields, including structural health monitoring, structural design optimization, multi-body dynamics, FE model updating, and experimental-FE model correlation [12–22].

The pioneering work for dynamic condensation methods is the static condensation method proposed by Guyan [2] and Irons [3] in the 1960 s. In 1989, O'Callahan [5] developed the improved reduced system (IRS) method employing the newly derived transformation matrix. This is calculated by adding the extra term containing inertial effect to Guyan's transformation matrix [2]. Since then, there have been considerable efforts to improve the solution accuracy of the IRS method. Friswell [7] developed the iterative IRS transformation matrix, and proposed the iterated IRS (IIRS) method. After that, Xia and Lin [8] proposed the modified IIRS transformation matrix, and improved the convergence speed of the IIRS method. To improve the computational efficiency of the Guyan, IRS, and IIRS methods, there have been several studies [23–28] employing physical domain based substructuring, which is a key concept of the component mode synthesis (CMS) methods [29–37].

Because the formulations of the IRS and IIRS methods are simple and produce accurate reduced models, those methods have been widely used. However, the IRS and IIRS methods have a critical limitation to handle very large FE models with hundreds of thousands of degrees of freedom (DOFs). This is because the part of the transformation matrix corresponding to the truncated DOFs is highly populated, which induces huge computational cost. Considering the recent trend of increase in the size of FE models, it is very important to overcome this limitation of the IRS and IIRS methods.

Recently, to resolve the limitation of the IRS method, we developed a very efficient and accurate method, which is named "algebraic dynamic condensation method" [1], exploiting the algebraic substructuring technique [38–43]. It was reported that the algebraic dynamic condensation method could handle a very large FE model with hundreds of thousands of DOFs, which could not be solved using the IRS method, and that the performance of this method was much superior to the IRS method in terms of both the solution accuracy and computational efficiency.

In this study, as an extension of the algebraic dynamic condensation method [1], a new iterative reduced-order modeling method is proposed. Using the algebraic substructuring technique [38], the global mass and stiffness matrices are automatically partitioned into small submatrices. To construct an initial reduced model,

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the substructural stiffness is condensed into the interface boundary, and the interface boundary is reduced using the dominant interface normal modes. Then, the initial reduced model is iteratively updated until satisfying the designated error tolerance through the iterative substructural inertial effect condensation procedure. To reduce computational cost, the formulation of the iterative reduced model is expressed by simple multiplications and summations of small submatrices. The performance of the proposed method is verified considering several large structural FE models. It is observed that the computational efficiency of the proposed method is much superior to that of the IIRS method, with more accurate solutions. Furthermore, the proposed method can handle large FE models that the IIRS method cannot handle.

In the following sections, the formulation of the IIRS method is reviewed briefly, and the proposed method is derived. We then evaluate the performance of the proposed method compared to that of the IIRS method using several structural problems. Finally, conclusions are drawn.

2. Iterated IRS (IIRS) method

In structural dynamics, the equations of motion for un-damped free vibration without damping are given by

$$\mathbf{M}_g\ddot{\mathbf{u}}_g + \mathbf{K}_g\mathbf{u}_g = \mathbf{0}, \tag{1}$$

where $\mathbf{M}_g$ and $\mathbf{K}_g$ are the global mass and stiffness matrices, respectively, and $\mathbf{u}_g$ is the global displacement vector. In the IIRS method, before reducing the global system, the global matrices and vector are separated as

$$\mathbf{M}_g = \begin{bmatrix} \mathbf{M}_t & \mathbf{M}_{tr} \\ \mathbf{M}_{tr}^T & \mathbf{M}_r \end{bmatrix},\ \mathbf{K}_g = \begin{bmatrix} \mathbf{K}_t & \mathbf{K}_{tr} \\ \mathbf{K}_{tr}^T & \mathbf{K}_r \end{bmatrix},\ \mathbf{u}_g = \begin{bmatrix} \mathbf{u}_t \\ \mathbf{u}_r \end{bmatrix}, \tag{2}$$

in which the subscripts t and r denote the truncated and retained DOFs, respectively, and the subscript tr denotes the coupled DOFs between t and r.

The global eigenvalue problem is defined by

$$\mathbf{K}_g\mathbf{u}_g = \lambda\mathbf{M}_g\mathbf{u}_g, \tag{3}$$

and its partitioned form is expressed as

$$\begin{bmatrix} \mathbf{K}_t & \mathbf{K}_{tr} \\ \mathbf{K}_{tr}^T & \mathbf{K}_r \end{bmatrix}\begin{bmatrix} \mathbf{u}_t \\ \mathbf{u}_r \end{bmatrix} = \lambda\begin{bmatrix} \mathbf{M}_t & \mathbf{M}_{tr} \\ \mathbf{M}_{tr}^T & \mathbf{M}_r \end{bmatrix}\begin{bmatrix} \mathbf{u}_t \\ \mathbf{u}_r \end{bmatrix}, \tag{4}$$

in which λ denotes the eigenvalue of the global system.

Expanding the first row in Eq. (4), the truncated DOFs vector $\mathbf{u}_t$ is written as

$$\mathbf{u}_t = -\mathbf{K}_t^{-1}\mathbf{K}_{tr}\mathbf{u}_r + \lambda\mathbf{K}_t^{-1}(\mathbf{M}_{tr}\mathbf{u}_r + \mathbf{M}_t\mathbf{u}_t). \tag{5}$$

Assuming a transformation matrix $\mathbf{T}$ between $\mathbf{u}_t$ and $\mathbf{u}_r$, the truncated DOFs vector $\mathbf{u}_t$ is rewritten as

$$\mathbf{u}_t = \mathbf{T}\mathbf{u}_r. \tag{6}$$

Substituting Eq. (6) into $\mathbf{u}_t$ in the right-hand side of the Eq. (5), the following equation is obtained

$$\mathbf{u}_t = [-\mathbf{K}_t^{-1}\mathbf{K}_{tr} + \lambda\mathbf{K}_t^{-1}(\mathbf{M}_{tr} + \mathbf{M}_t\mathbf{T})]\mathbf{u}_r, \tag{7}$$

and from the relation $\mathbf{u}_t = \mathbf{T}\mathbf{u}_r$ in Eq. (6), the transformation matrix $\mathbf{T}$ can be defined as

$$\mathbf{T} = \mathbf{T}_s + \lambda\mathbf{K}_t^{-1}(\mathbf{M}_{tr} + \mathbf{M}_t\mathbf{T}) \ \text{ with } \mathbf{T}_s = -\mathbf{K}_t^{-1}\mathbf{K}_{tr}. \tag{8}$$

Using the transformation matrix $\mathbf{T}$ in Eq. (8), the global displacement vector $\mathbf{u}_g$ is represented by

$$\mathbf{u}_g = \begin{bmatrix} \mathbf{u}_t \\ \mathbf{u}_r \end{bmatrix} = \begin{bmatrix} \mathbf{T} \\ \mathbf{I}_r \end{bmatrix}\mathbf{u}_r = (\mathbf{T}_G + \lambda\mathbf{T}_a)\mathbf{u}_r \ \text{ with } \mathbf{T}_G = \begin{bmatrix} \mathbf{T}_s \\ \mathbf{I}_r \end{bmatrix}, \quad \mathbf{T}_a = \begin{bmatrix} \mathbf{K}_t^{-1}(\mathbf{M}_{tr} + \mathbf{M}_t\mathbf{T}) \\ \mathbf{0} \end{bmatrix}, \tag{9}$$

where $\mathbf{T}_G$ is the Guyan transformation matrix [2], which is sometimes called the "static condensation matrix", $\lambda\mathbf{T}_a$ is an additional transformation matrix containing the inertial effects of the truncated DOFs, and $\mathbf{I}_r$ is the identity matrix for the retained DOFs.

Considering only $\mathbf{T}_G$ in Eq. (9), the global displacement vector $\mathbf{u}_g$ is approximated as

$$\mathbf{u}_g \approx \bar{\mathbf{u}}_g = \mathbf{T}_G\mathbf{u}_r, \tag{10}$$

and applying Eq. (10) into Eq. (3), the following reduced eigenvalue problem is obtained

$$\bar{\mathbf{K}}_G\mathbf{u}_r = \bar{\lambda}\bar{\mathbf{M}}_G\mathbf{u}_r \quad \text{with } \bar{\mathbf{M}}_G = T_G^T\mathbf{M}_g\mathbf{T}_G,\ \bar{\mathbf{K}}_G = \mathbf{T}_G^T\mathbf{K}_g\mathbf{T}_G, \tag{11}$$

in which $\bar{\mathbf{M}}_G$ and $\bar{\mathbf{K}}_G$ are the reduced mass and stiffness matrices in the Guyan reduction, and $\bar{\lambda}$ is the approximated eigenvalue.

Pre-multiplying $\bar{\mathbf{M}}_G^{-1}$ to Eq. (11), we can obtain the following equation

$$\bar{\lambda}\mathbf{u}_r = \mathbf{H}_G\mathbf{u}_r \ \text{ with } \mathbf{H}_G = \bar{\mathbf{M}}_G^{-1}\bar{\mathbf{K}}_G. \tag{12}$$

In Eq. (9), using $\bar{\lambda}$ instead of λ, and considering the relation $\bar{\lambda}\mathbf{u}_r = \mathbf{H}_G\mathbf{u}_r$ in Eq. (12), the approximated global displacement vector $\bar{\mathbf{u}}_g$ is redefined as

$$\bar{\mathbf{u}}_g = \mathbf{T}_1\mathbf{u}_r \ \text{ with } \mathbf{T}_1 = \begin{bmatrix} \mathbf{T} \\ \mathbf{I}_r \end{bmatrix},\ \mathbf{T} = \mathbf{T}_s + \mathbf{K}_t^{-1}(\mathbf{M}_{tr} + \mathbf{M}_t\mathbf{T})\mathbf{H}_G. \tag{13}$$

Note that it is not possible to directly calculate the transformation matrix $\mathbf{T}_1$, because the matrix $\mathbf{T}$ is implicit in the formulation. Therefore, an iterative scheme needs to be employed to calculate the transformation matrix $\mathbf{T}_1$.

Employing an iterative scheme, we can define an iterative transformation matrix $\mathbf{T}_1^{(k)}$ as

$$\mathbf{T}_1^{(k)} = \begin{bmatrix} \mathbf{T}^{(k)} \\ \mathbf{I}_r \end{bmatrix} \ \text{with } \mathbf{T}^{(k)} = \mathbf{T}_s + \mathbf{K}_t^{-1}(\mathbf{M}_{tr} + \mathbf{M}_t\mathbf{T}^{(k-1)})\mathbf{H}^{(k-1)} \quad \text{for } k \geqslant 2, \tag{14}$$

in which

$$\mathbf{H}^{(k-1)} = (\bar{\mathbf{M}}^{(k-1)})^{-1}\bar{\mathbf{K}}^{(k-1)},\ \bar{\mathbf{M}}^{(k-1)} = (\mathbf{T}_1^{(k-1)})^T\mathbf{M}_g\mathbf{T}_1^{(k-1)},\ \bar{\mathbf{K}}^{(k-1)} = (\mathbf{T}_1^{(k-1)})^T\mathbf{K}_g\mathbf{T}_1^{(k-1)}, \tag{15a}$$

$$\mathbf{T}^{(1)} = \mathbf{T}_s,\ \mathbf{H}^{(1)} = \mathbf{H}_G,\ \mathbf{T}_1^{(k)} = \mathbf{T}_G,\ \bar{\mathbf{M}}^{(1)} = \bar{\mathbf{M}}_G,\ \bar{\mathbf{K}}^{(1)} = \bar{\mathbf{K}}_G, \tag{15b}$$

where the superscript k denotes the k^{th} iteration, and when $k = 2$, $\mathbf{T}_1^{(k)}$ is equivalent to the transformation matrix of the improved reduce system (IRS) method [5].

Thus, using the transformation matrix $\mathbf{T}_1^{(k)}$ in Eq. (14), the reduced mass and stiffness matrices in the IIRS method are obtained by

$$\bar{\mathbf{M}}_g = (\mathbf{T}_1^{(k)})^T\mathbf{M}_g(\mathbf{T}_1^{(k)}),\ \bar{\mathbf{K}}_g = (\mathbf{T}_1^{(k)})^T\mathbf{K}_g(\mathbf{T}_1^{(k)}), \tag{16}$$

and the reduced eigenvalue problem is given by

$$\bar{\mathbf{K}}_g(\bar{\boldsymbol{\varphi}})_{\mathbf{i}} = \bar{\lambda}_{\mathbf{i}}\bar{\mathbf{M}}_g(\bar{\boldsymbol{\varphi}})_{\mathbf{i}} \quad \text{for } i = 1, 2, \cdots, N_r, \tag{17}$$

where $\bar{\lambda}_i$ and $(\bar{\boldsymbol{\varphi}})_i$ are the approximated eigenvalues and eigenvectors in the IIRS method, respectively, and N_r is the number of the retained DOFs. Herein, the i^{th} approximated global eigenvector $(\bar{\boldsymbol{\varphi}}_g)_i$ can be calculated by $(\bar{\boldsymbol{\varphi}}_g)_i = \mathbf{T}_1^{(k)}\bar{\boldsymbol{\varphi}}_i$.

It is important to note that, when a large size FE model (over 10^5 DOFs) is considered, the IIRS method could induce a huge computational cost due to a highly populated matrix $\mathbf{T}$ in Eq. (13) requiring large computer memory, which is updated through the iterative procedure to construct the iterative