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Maximizing spatial decay of evanescent waves in phononic crystals by topology optimization

Yafeng Chen[a], Xiaodong Huang[a,b,*], Guangyong Sun[a], Xiaolei Yan[c], Guangyao Li[a]

[a] Key Laboratory of Advanced Technology for Vehicle Body Design & Manufacture, Hunan University, Changsha 410082, China
[b] Centre for Innovative Structures and Materials, School of Engineering, RMIT University, GPO Box 2476, Melbourne 3001, Australia
[c] School of Mechanical and Automobile Engineering, Fujian University of Technology, Fuzhou 350108, China

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ABSTRACT

The propagation of evanescent waves inside phononic band gaps is important for the design of phononic crystals with desirable functionalities. This paper extends the bi-directional evolutionary structure optimization (BESO) method to the design of phononic crystals for maximizing spatial decay of evanescent waves. The optimization objective is to enlarge the minimum imaginary part of wave vectors at a specified frequency. The study is systematically conducted for both out-of-plane and in-plane waves at various frequencies. Numerical examples demonstrate that the proposed optimization algorithm is effective for designing phononic crystals with maximum spatial decay of evanescent waves. Various topological patterns of optimized phononic crystals are given. The results also show that the proposed optimization algorithm can successfully overcome difficulties in opening band gap at desirable frequencies, especially for the complete band gap and multiple band gaps.

1. Introduction

Phononic crystals are usually composites made of one material periodically embedded in others with different physical properties (elasticity modulus and mass density) [1]. One of the most important features of phononic crystals is the possible existence of band gaps, within which the propagation of waves is prohibited and only evanescent waves with spatial decay are existed. Phononic band gap crystals have a wide range of potential applications in sound insulation [2–5], defect based energy trapping [6–9], acoustic filtering [10–12], and wave guiding [13–16].

To analyze the dynamic behavior of phononic crystals, it is necessary to calculate band diagram by solving the dispersion equation relating the wave vector $\mathbf{k}$ to its frequency ω. The traditional way is to consider given Bloch wave vectors, $\mathbf{k}$, within the first Brillouin zone and solve for the frequencies, ω, of propagating waves. The resulting band diagram based on the $\omega(\mathbf{k})$ method identifies the band gaps within which no band exists [17]. Since wave propagation is inhibited within a band gap, only evanescent Bloch waves are left to explain the exponentially decreasing transmission of waves. Thus, the classic band diagrams for only real wave vectors are of no help [18]. To describe the characteristics of evanescent waves, it is necessary to compute the complex band diagram by solving for wave vectors with given frequencies. This approach is known as the $\mathbf{k}(\omega)$ method. Laude et al. [18] employed the extended plane-wave expansion (EPWE) to obtain complex band diagrams. Romero et al. [19–24] conducted a serial of theoretical and experimental study on the evanescent Bloch waves in phononic crystals. They confirmed that the occurrence of a bandgap is not indicated by an absence of bands but by the evanescent character of Bloch waves, and the band gap is defined with a range of frequencies where all waves must be evanescent. The spatial decay inside the band gap is multi-exponential and dominated by the minimum imaginary part of wave vectors.

Since physical properties of phononic crystals highly depend on the spatial distribution of the constituent materials, topological design of phononic crystals has aroused a growing attention in recent years [25]. Sigmund and Jensen first conducted the optimization of phononic band gap crystals by combining finite element method (FEM) with the method of moving asymptotes (MMA) [26]. Thereafter, genetic algorithm (GA) and gradient-based topology optimization, in conjunction with the fast plane wave expansion method (FPWE) or FEM, were developed to maximize the relative or absolute band gap width of phononic crystals [27–33]. Li et al. [34,35] developed the bi-directional evolutionary structural optimization (BESO) method for maximizing band gaps

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[*] Corresponding author at: Centre for Innovative Structures and Materials, School of Engineering, RMIT University, GPO Box 2476, Melbourne 3001, Australia.

E-mail address: huang.xiaodong@rmit.edu.au (X. Huang).

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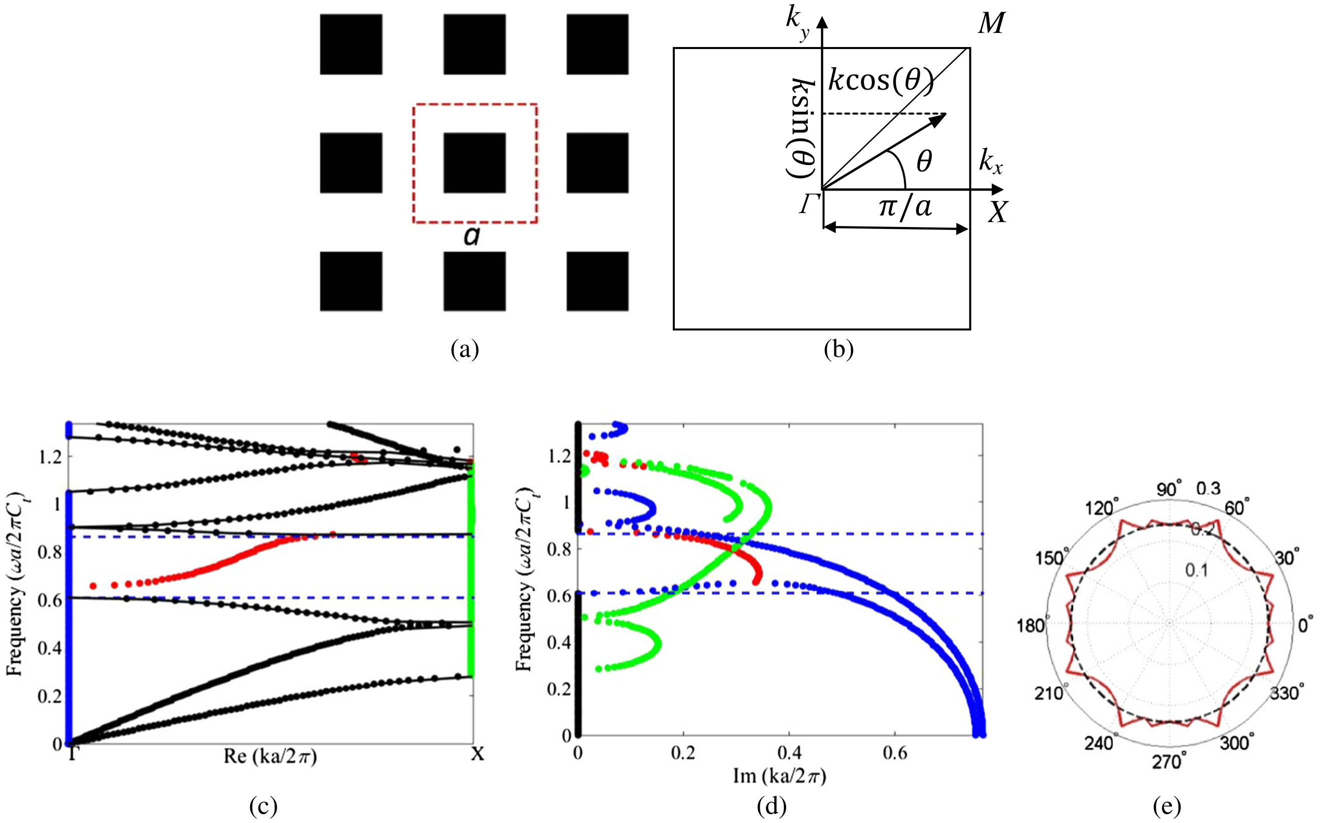
for in-plane and out-of-plane waves based on the $\omega(\mathbf{k})$ method. In those studies, phononic crystals are assumed to be infinite and arranged periodically along the plane and the spatial decay of evanescent waves within band gaps is totally neglected. Utilizing the viscoelastic properties of materials, Andreassen et al. [36] presented topology optimization of viscoelastic phononic crystals by maximizing the attenuation factor and identified the possible formation of band gaps. However, topology optimization of elastic phononic crystals for spatial decay of evanescent waves based on the $\mathbf{k}(\omega)$ method has never been investigated.

Early studies on topology optimization of phononic band gap crystals possibly encountered difficulties in opening band gaps [26]. Even successful, the optimization can only maximize a band gap between the specified adjacent bands, and therefore hardly obtain the band gap at a desirable frequency, especially for the complete band gap for both out-of-plane and in-plane waves and multiple band gaps [34]. Furthermore, evanescent waves within band gaps may still propagate through finite phononic crystals, if spatial decay of evanescent waves is not large enough. Therefore, it is of significance to maximize spatial decay of evanescent waves at a specified frequency, and expected to obtain the band gap at a desirable frequency and overcome the difficulties in opening band gaps in the traditional band gap optimization.

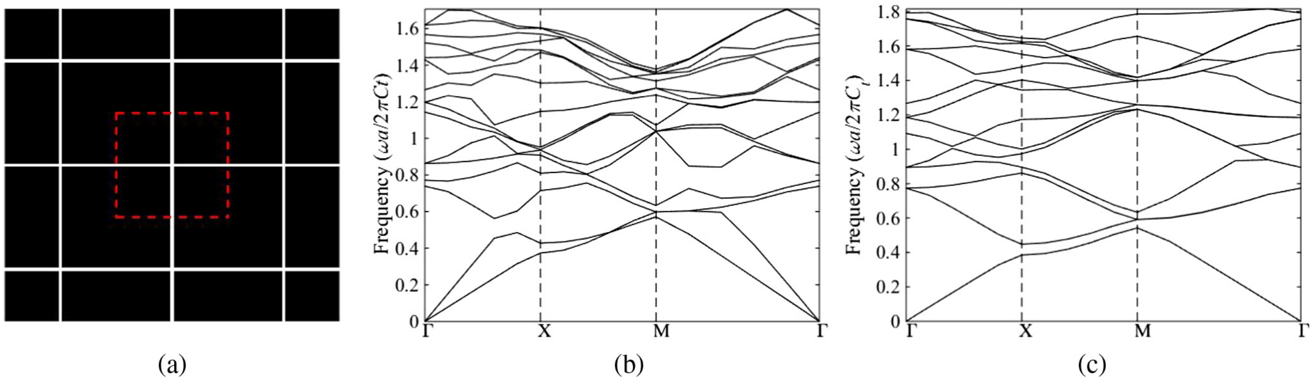
This paper will extend BESO algorithm to maximize the imaginary part of wave vectors under a single or multiple frequencies. The rest of the paper is organized as follows. Section 2 introduces



(a) (b)

(c) (d) (e)

Fig. 1. (a) The 3 × 3 primitive unit cells of a phononic crystal (black-tungsten, white-aluminum). (b) Illustration of the propagation direction specified by the angle θ and the irreducible Brillioun zone (Γ-X-M-Γ). (c) Real part of the complex band diagram. (d) Imaginary part of the complex band diagram. (e) The minimum decay contour at $\omega a/2\pi C_t = 0.7$.



(a) (b) (c)

Fig. 2. Initial design and its classic band diagrams: (a) 3 × 3 primitive unit cells (black-tungsten, white-aluminum); (b) in-plane wave and (c) out-of-plane wave.