Computers and Structures 182 (2017) 504–525

Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

Numerical characterisation of uncured elastomers by a neural network based approach

C. Zopf, M. Kaliske *

Institute for Structural Analysis, Technische Universität Dresden, 01062 Dresden, Germany

ARTICLE INFO

Article history:
Received 2 August 2016
Accepted 30 December 2016
Available online 26 January 2017

Keywords:
Model-free material characterisation
Recurrent neural network
Micro-sphere approach
Uncured rubber material
Finite element forming simulation

ABSTRACT

This research paper contributes to the model-free characterisation of elastic and inelastic materials. The introduced material description is based on neural networks for the constitutive stress-strain-relationship and is an alternative to a classical constitutive material description. This approach is an efficient method to represent the material behaviour numerically. The performance of the novel model-free formulation is exemplary shown for uncured natural rubber. One major advantage of the presented approach is the capability to use it for the representation of a wide range of materials.

The novel model-free characterisation consists of a neural network that is coupled to the so-called micro-sphere approach. This formulation was developed for the characterisation of rubber like materials and takes the micro-structure of the material into account. As a benefit of this coupling, the neural networks, representing the stress-stretch-dependency, have to be exploited analysed only in onedimensional direction. In the first instance, the derivation is introduced for an **A**rtificial **N**eural **N**etwork to yield a pure elastic description. Subsequently, the model-free approach is expanded in order to represent inelastic material behaviour as well. This extended formulation is obtained via a **R**ecurrent **N**eural **N**etwork.

Finally, uncured natural rubber material is described by the derived numerical approach. The model-free characterisation is validated by the finite element simulation of material tests. A complex forming of a rubber block into a mould is basis for a final validation of the model-free description.

1. Introduction

For the development of a classical material description, several steps are required. Firstly, extensive tests of the material to be modelled are conducted. Subsequently, an appropriate rheology, that captures the behaviour has to be chosen. Based on the rheology, stress and material tensors have to be derived out of the constitutive equations. Finally, the material parameters have to be identified and the material characterisation is verified and validated. This stepwise procedure consists of two approximations of the material behaviour. The first approximation is made while choosing the rheology and the accompanying constitutive equations. The second approximation is carried out by the parameter identification for the material model.

The major advantages of a model-free approach are numerical efficiency, reduction of the number of approximations to one and generality to use this principal numerical formulation for different materials without changing it. The model-free approach directly maps the strain to the stress without construction of complex constitutive descriptions. No internal, local iteration is executed, like it is performed for classical inelastic material models to determine internal variables. Therefore, the model-free approach yields the total stresses in dependency on the strain, which leads to an enormous reduction of computational cost. Inelastic phenomena are included in the presented model-free characterisation. While using the model-free formulation, it is not required to develop a rheological model. Here, material test results are used to identify the parameters of the numerical formulation, to reduce the description only to one approximation. Additionally, this model-free formulation can be used for a wide range of different materials.

Commonly, the model-free material characterisation is based on **A**rtificial **N**eural **N**etworks (see [1] or [2]). For the development of the ANN, the human brain serves as a template as it consists of neurons which are connected to each other via synaptic links. ANN are adaptive and, therefore, are able to train and to map different functional dependencies. The advantage of the ANN is that it reliably approximates very complex and highly nonlinear data. Due to their performance and their capabilities, it is possible to use the ANN within a finite element simulation as a model-free

* Corresponding author.
E-mail address: michael.kaliske@tu-dresden.de (M. Kaliske).

http://dx.doi.org/10.1016/j.compstruc.2016.12.012
0045-7949/

material characterisation. In [3,4], the ANN is used to map strains onto stresses. This mapping is introduced for small strain theory. For the mapping of inelastic material behaviour in [3,5–7], different approaches are presented. All these formulations use the ANN for the model-free material characterisation. In order to yield a path dependent material description, networks are introduced, that have a nested architecture or define output parameters of the ANN in the previous time step as an input parameter of the current time step. In this publication, **R**ecurrent **N**eural **N**etworks are used for the inelastic case instead of the ANN. The RNN has a similar architecture like the ANN but it contains additionally so-called context neurons. The context neurons are updated in each time step and the updated information is stored until the next time step. This procedure leads to a delay in information flow in the network within the time step and, therefore, to a path dependent behaviour of the RNN. In [8–10], the RNN is used in order to model inelastic material behaviour instead of using a constitutive model. The mentioned model-free approaches are developed for small strain. An extension to finite strain theory is presented in [11]. The disadvantage of the mentioned approaches is the fact, that the use of the RNN beyond the range of training data is not permissible. In order to capture all relationships between the entries of the strain and the stress tensor, material tests which consider different states of loading are required for the training. The consideration of only few standardised (uniaxial, equibiaxial or shear) material tests is insufficient for the training, but material tests which capture a combination of these tests have to be developed. This yields expensive and time dependent material test procedures.

In order to reduce the number and the complexity of the required material tests, a novel approach is introduced in this paper. Here, the RNN is coupled to the micro-sphere framework (see [12]). The micro-sphere description is developed for modelling of rubber like materials and takes the micro-structure of the material into account. The stresses at the material point are given in a uniaxial continuous distribution of the orientation in space. For this approach, the chains are distributed according to the surface of a sphere. This coupling leads to a onedimensional analysis of the RNN and, therefore, only uniaxial tensile and compression tests are required for training and validation. This method leads to an enormous reduction of expensive material testing and an easier training of the network. In addition, a finite element simulation, which is based on the novel model-free material characterisation, is more stable, because the onedimensional RNN behaves more smoothly in comparison to an RNN that has to map a threedimensional state of strain.

In the next section, the essential parameters of the material test procedure are presented. Subsequently, the micro-sphere model as well as the ANN and RNN are introduced. The derivation of the novel model-free material characterisation is conducted for pure elastic material in a first step. Here, the ANN is coupled to the micro-sphere model. Then, the ANN is replaced by the RNN which enables a representation of inelastic material behaviour by the model-free formulation. For the challenging training of the RNN, which is coupled to the micro-sphere framework, three different options are proposed. A satisfying comparison between the simulated and measured material tests shows that the numerical implementation of the programme code and the training of the RNN are carried out properly. At the end, a complex moulding simulation is developed, which shows the performance of finite element simulations under consideration of the novel model-free material representation.

2. Properties of uncured elastomers

The numerical characterisation of material requires fundamental knowledge about its constitutive behaviour and its determining properties. This knowledge is obtained by extensive material tests. Especially, for the representation of uncured elastomers with the proposed model-free approach, a wide range of material tests has to be used. In this article, the functionality and the performance of the model-free approach are shown for an uncured natural rubber compound, which is presented in [13]. Based on the material tests in [13], the behaviour and the properties of the uncured natural rubber compound are described in detail.

Within the test procedure, uniaxial and pure shear tests are carried out. Only the uniaxial material test results are considered for the determination of the unknown parameters of the model-free formulation. The pure shear test is used in order to validate the developed characterisation. Hence, the focus of the material testing is on uniaxial material tests. For an appropriate experimental characterisation of the uncured rubber compound, tensile tests with one load cycle until elongation of break (single stage) and cyclic tensile tests are conducted. The results are shown in Fig. 1(a). The stretch rate for all tensile tests is $\dot{\bar{\lambda}} = 0.022\ \mathrm{s}^{-1}$. The cyclic tensile tests consist of 5 load cycles. In each load cycle, the maximum strain is increased by 20%. The stress-stretch-dependencies of the loading and unloading period are different. The differences are due to energy dissipation within the rubber material and, hence, the chosen description has to be able to reproduce inelastic behaviour.

As expected, the single stage tests represent the envelop of the cyclic experiment, but a small difference between both stress-stretch-dependencies is seen. Additionally, in Fig. 1(a) two single stage tensile tests are plotted in order to show, that the uncured elastomer material is associated with some variation. The overall difference in between the different tests is less than 10%. In a further step, these uncertainties may be taken into account (compare [14]). In here, uncertainty is neglected. The authors are very well aware, that it is not possible to capture the uncured rubber behaviour identically by the model-free approach. In order to obtain an approach that represents the uncured rubber behaviour realistically, test results, which represent the major aspects of the uncured rubber material are used for the identification of the parameters of the approach.

Additionally, relaxation tests as well as a special setting, which enables the separation of the total stretch into plastic and viscoelastic parts of deformation, are carried out. The relaxation tests consist of three different load steps ($\bar{\lambda} = 1.30, \bar{\lambda} = 1.60$ and $\bar{\lambda} = 1.90$) and a relaxation time of 900 s after each load step. The test, which identifies the plastic response, is comparable to the cyclic tensile test. In contrast to the cyclic test, the specimen is subjected to a Kirchhoff stress of $0.02\ \mathrm{N/mm^2}$ for 300 s after each unloading period. Within this time, the viscoelastic stretch disappears and only the irreversible part of deformation remains. It can be stated that for the uncured rubber, a permanent increase of the irreversible part of deformation is found. Uncured rubber material is in contrast to brittle materials not characterised by a yield surface, where the material behaves elasticly until the yield stress is reached. The ratio between viscoelastic and irreversible parts of deformation is 65–35% for the investigated uncured compound. In Fig. 1(b), the results of the relaxation and the plastic response tests are shown.

In addition to the tensile tests, two different kinds of uniaxial compression tests are conducted with a single stage test until a compression stretch of $\bar{\lambda} = 0.25$ and, additionally, a cyclic compression test. The cyclic experiment consists of seven load cycles. In each load cycle, the maximum stretch is changed by $\Delta\bar{\lambda} = 0.10$. Analogously to the tensile tests, the single stage compression test represents the envelop of the cyclic one (compare Fig. 2(a)). The pure shear results are used for validation. The pure shear experiment is a tensile test with a very wide specimen. The ratio between the initial clamping length and the transverse length