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The finite element method with overlapping elements – A new paradigm for CAD driven simulations ☆

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ABSTRACT

A major difficulty in finite element analysis is the preparation of an effective mesh leading to a good response solution. In engineering analyses of complex components, oftentimes, much more time is spent to arrive at an adequate mesh than to obtain the solution of the established finite element model. The difficulty in meshing is due to the fact that finite elements need to abut each other and cannot overlap. This can lead to highly distorted elements, e.g. sliver elements, reducing the accuracy of solution. In practice, to establish an effective mesh, frequently, significant expertise in building meshes is needed, because great care must be taken in cleaning up the CAD geometry and preparing an effective mesh.

The objective in this paper is to present a new finite element solution scheme including meshing in which the elements can overlap. The property that finite elements can overlap removes many of the meshing difficulties, leads to an effective meshing procedure and an overall easy-to-use solution scheme for an analyst or a designer.

We first present the meshing scheme that we propose, which combines the use of traditional finite elements and overlapping finite elements. A particular feature is that the meshing procedure can be directly embedded in CAD driven solutions. We then present the theory used for the formulation of the overlapping finite elements and the coupling with traditional finite elements. We consider spherical and brick-shaped overlapping finite elements for which the theory is largely based on the formulation of the method of finite spheres. Finally, we illustrate the complete solution scheme in the analysis of some two-dimensional problems using the CAD geometry as the starting point.

While the paper presents a new paradigm for analysis in CAD environments, with much potential, we realize that much further study and research is needed on some of the important ingredients of the method to render the complete procedure effective for general practical engineering analyses of static and dynamic problems.

1. Introduction

The finite element method is now established as an effective procedure to simulate on the computer the behavior of structures. Quite general structures can be analyzed, from large scale to very small scale structures, such as from large bridges, to motor cars, to DNA structures [1,2]. However, in all finite element simulations, it is necessary to establish an appropriate and effective mesh of elements, which may require a large effort for the analyst. Since also experience is needed to construct an adequate mesh, we see that mostly only experienced analysts can perform an effective simulation, even in linear analysis. The difficulties of obtaining an adequate and hence good mesh should ideally be removed from the analysis process. If this is achieved, the finite element method will be much more employed, notably by designers in the CAD environment.

Since there are the difficulties of meshing, many meshless methods have been designed, see Refs. [3–9] and the references therein. Much research effort has been expended to develop an effective meshless method. Nevertheless, all reliable meshless methods (that do not entail the adjustment of numerical factors [1]) have been shown to be numerically expensive for practical usage when compared with the traditional finite element method [10–15] and, while the overall aim of using meshless methods is very attractive, such methods have not yet found broad use in engineering practice.

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The objective in this paper is to present a solution scheme that uses largely traditional finite elements in uniform mesh layouts in the interior of the analysis domain and overlapping finite elements near the boundaries to cover the complete analysis domain. The primary aim is to reduce the time and effort spent on meshing.

An approach using "overlapping grids" in finite difference solutions (the Chimera approach) has been developed and used for some time in fluid dynamics, see Refs. [16–20]. Here finite difference grids are superimposed at their boundaries to cover the complete analysis domain. Our approach is related to those schemes but with the important distinction that we use finite elements and overlapping finite elements with much more generality in direct coupling to CAD geometries for meshing complex geometries.

We first present the meshing scheme with some emphasis on how it can be used effectively in CAD driven analyses. Then we summarize the theory of the overlapping finite elements, which can be spherical and brick-shaped elements, including how the coupling to traditional elements is reached.

To indicate the accuracy obtained and resulting computational expense when only using overlapping finite elements, we consider briefly the method of finite spheres without traditional finite elements [21–23]. This discussion shows the potential of using overlapping finite elements.

Next we illustrate the complete analysis procedure based on the new proposed meshing scheme in the solution of some two-dimensional problems. The response is reached by automatically discretizing the surface (boundary) of the CAD part, automatic meshing and the solution. This illustration shows the potential of the complete analysis approach.

The paper is expanding on ideas presented earlier in Ref. [24] (from which some text is used) giving new theoretical details and results. We present a new analysis paradigm, the general theory, and results of some simple analysis problems. However, as concluded in the paper, considerable research and development effort is still needed to study and further develop the meshing and analysis scheme in order to fully harvest the potential of the new approach.

2. An effective way of meshing using traditional and overlapping finite elements

Considering the requirements of a reliable and generally applicable analysis procedure [1], the method of finite spheres can be regarded as one of the most effective meshless methods available. Like all meshless methods, it was designed to reduce the time of preparing a numerical model for a given physical problem, namely, the time and effort spend on meshing. However, while having some good attributes, the method is in most cases numerically too expensive to use.

Our objective in this section is to propose that overlapping elements – the spheres in the method of finite spheres being one type of such elements – be used effectively with very simple Cartesian meshes of traditional finite elements.

To explain the novel approach of using overlapping finite elements, consider the geometry of a two-dimensional part generated using a CAD software, like SolidWorks, as shown in Fig. 1. The discretization would be performed as follows.

The first step is to generate a two-dimensional grid over the whole part, with Δx and Δy as distances between the lines, see Fig. 2(a). An algorithm would automatically calculate the spacing depending on the geometry, or the analyst would specify the spacing. The part can now be thought of as being 'immersed' in a grid, which can obviously be established with negligible human and computational effort.

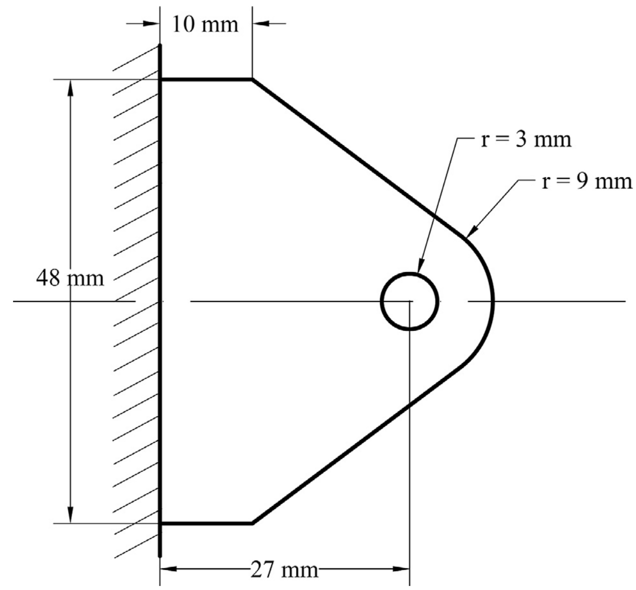

Fig. 1. A typical CAD part analyzed.

The second step is that "straight line segments" Δs are used for discretizing the boundary of the part. This segment length is best established automatically by an algorithm using the geometry of the CAD part and in general will vary in size along the boundary. Here, Δs should be small enough so that the segment lines will represent the complete boundary of the part with sufficient accuracy, see Fig. 2(b).

However, the segment length Δs can also be such that small imperfections in the CAD geometry, resulting from the complexities of the boundary representation using CAD procedures, are ignored and not included in the analysis. Some such cases leading to a non-watertight geometry are shown in Fig. 3. An important ingredient in the scheme is that Δs can vary along the boundary and can be selected automatically to discretize the boundary ignoring imperfections.

The third step is that all Cartesian cells that do not cut the numerical boundary established in step 2 are represented by traditional finite elements, like 4-node elements. The other cells are removed, see Fig. 2(c), and hence some empty space is generated.

The fourth step is that the boundary is meshed with overlapping finite elements using the segment length as spacing. It is important to place the centers of these elements at the boundary points (the end points of the boundary segment lines) because then the displacement boundary conditions can be easily imposed [7,22]. These overlapping finite elements must extend over to the traditional finite elements (established in step 3) to fill in all empty space and provide for the coupling of the traditional and overlapping elements. If the one layer of spheres placed along the boundary does not sufficiently extend into the traditional finite elements, additional overlapping elements are placed. We illustrate this process generically in Fig. 2(d).

In three-dimensional analysis, the same steps are followed but the grid is for the three Cartesian coordinate directions and triangular surfaces of varying size are used to discretize the surface boundary of the CAD part in step 2. Then traditional brick finite elements are employed in conjunction with overlapping brick or spherical elements.

The coupling between the overlapping finite elements and the traditional finite elements is achieved as presented in Section 3.2.

The effort in meshing using this approach is much smaller than using traditional finite elements throughout the analysis domain.