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Shape optimization and optimal control for transient heat conduction problems using an isogeometric approach

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ABSTRACT

This work is concerned with the development of a framework to solve shape optimization problems for transient heat conduction problems within the context of isogeometric analysis (IGA). A general objective functional is used to accommodate both shape optimization and passive control problems under transient conditions. An adjoint sensitivity analysis, which accounts for possible discontinuities in the objective functional, is performed analytically and subsequently discretized within the context of IGA. The gradient of the objective functional is used in a descent algorithm to solve optimization problems. Numerical examples are presented to validate and demonstrate the capacity to manage thermal fields under transient conditions.

1. Introduction

Engineering devices used for thermal management in applications where the temperature and/or the heat exchanged play an important role, such as heat exchangers, cooling components, heat sinks or thermal protection layers, are typically designed to control the maximum or minimum temperature that a system should be exposed to or to guarantee a certain heat exchange rate. Given some performance requirements, a judicious choice of materials and shapes is made and the original design is subsequently optimized to improve its performance. This optimization process is typically based on steady-state conditions. Nevertheless, in many technologically-relevant applications, thermal conditions fluctuate during operation. In these cases, an active control system is sometimes used for thermal management. The drawback of an active control system is its additional cost, which may hinder its usefulness. An attractive alternative is passive control, which relies on a suitably-designed system that takes a priori into account fluctuations in the thermal fields. In particular, passive control may be achieved through a shape optimization procedure that finds an optimal shape under transient heat conduction conditions.

Shape optimization has received renewed attention due to development of Isogeometric Analysis (IGA) as an alternative to the (classical) finite-element method (FEM) [1]. The main advantage of IGA is that the numerical analysis is carried out with the same shape functions used in most commercially-available Computer-Aided Design (CAD) programs. Consequently, the bottleneck of converting a CAD-generated design into an FEM-ready mesh is avoided [2]. CAD software typically uses non-uniform rational B-splines (NURBS) [3] to represent the design geometry. In fact, prior to the development of IGA, shape optimization has also relied on NURBS for design purposes (see, e.g., Braibant and Fleury [4], Espath et al. [5] and Wang and Zhang [6]). More recently, the integration of IGA and NURBS-based shape optimization has been exploited to improve the overall efficiency and accuracy of the method, particularly in terms of an unified design and analysis parametrization that provides an enhanced sensitivity analysis (see, e.g., Wall et al. [7], Qian [8]).

Isogeometric shape optimization has been used for curved beam structures in Nagy et al. [9] and Nagy et al. [10], vibrating membranes in Manh et al. [11], fluid mechanics in Nørtoft and Gravesen [12], pulsatile ventricular assist devices in Long et al. [13], shells in Nagy et al. [14], Kiendl et al. [15] and Ha [16], photonic crystals in Qian and Sigmund [17], composite fiber orientation in [18], heat conduction problems in Qian and Sigmund [19], Stokes flow problems in Yoon et al. [20]. Isogeometric shape optimization using boundary element method is presented in Park et al. [21]. Isogeometric shape optimization using implicit differentiation method for shape sensitivity analysis is studied by Lian et al. [22–24] based on boundary element method. The aforementioned contributions illustrate the wide range of applications of the isogeometric shape optimization approach. However, the method has been limited to static or steady-state conditions. Recently, the isogeometric shape optimization method was extended in

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Wang and Turteltaub [25] to analyze quasi-static processes with slowly-varying external loads.

In the present work, one goal is to extend the isogeometric approach to carry out shape design and passive control of problems governed by a transient behavior, in particular transient heat conduction. Previous contributions related to inverse shape design work for transient heat conduction, which have relied mostly on a classical FEM analysis, can be found in Dulikravich [26], Jarny et al. [27], Huang and Chaing [28] and Korycki [29]. Shape sensitivity analysis for the linear or nonlinear transient heat conduction problems can be found in Haftka [30], Tortorelli and Haber [31], Tortorelli et al. [32], Służalec and Kleiber [33], Kleiber and Slużalec [34], Dems and Rousselet [35,36], Gu et al. [37] and Korycki [38]. Shape optimization work of thermoelastic problems with transient thermal field can be found in Kane et al. [39], Gao and Grandhi [40] and Song et al. [41]. Other related design sensitivity analysis may be found in Haftka and Grandhi [42], Van Keulen et al. [43], Choi and Kim [44] and Nanthakumar et al. [45].

A second goal in the present work is to extend the design sensitivity for situations where the objective functional is measured only in selected areas of the design region and during selected intervals of the design time period. This modeling feature is useful in cases where only portions of the design region and/or specific time intervals of the analysis period are relevant for the overall shape design. In that context, the sensitivity analysis requires an extended version of the transport theorems to account for discontinuities in the objective functional.

The structure of the present work is as follows: the general formulation of the shape optimal design and passive control considering the jump conditions in the objective functional is presented in a continuous setting in Section 2 (i.e., continuous description of the design and analysis spaces). By assigning a Lagrange multiplier for the strong form of the governing equations at every material point during the transient time interval, the continuous adjoint sensitivity analysis is developed in Section 3. Necessary details are included to treat the discontinuities of the characteristic functions that represent the regions (in space and/or time) where the objective functional is defined. The isogeometric analysis and design discretization is discussed briefly in Section 4. The design problem and its sensitivity are subsequently discretized in Section 5 within the context of IGA. This section also includes the algorithm used to numerically solve shape optimization problems. To validate the methodology, two benchmark problems, namely a minimum surface problem and a passive temperature control problem at predetermined time are presented in Section 6.1. To further illustrate the range of possible applications, two examples are presented in Sections 6.2 and 6.3, namely the shape design of a plunger for a molten glass forming die and a thermal protection system (TPS) for a re-entry ballistic vehicle nose. Finally, some concluding remarks are given in Section 7.

2. Problem statement

2.1. Design function and heat conduction problem

Consider an isotropic, homogeneous and linearly thermally-conducting material that occupies a region Ω^s with boundary Γ^s as shown in Fig. 1. The superscript s is a continuous scalar parameter that represents different configurations (or states) of the region. Each configuration corresponds to a design of the structure. The state of the region with $s = 0$, i.e. Ω^0, is called the referential or initial design. A material design point $\boldsymbol{p} \in \Omega^0$ is mapped to a position $\boldsymbol{x} = \hat{\boldsymbol{x}}[\boldsymbol{p}, s] \in \Omega^s$ by a design function $\hat{\boldsymbol{x}}$ as indicated in Fig. 1. For simplicity, the mapping $\hat{\boldsymbol{x}}$ is henceforth also denoted as $\boldsymbol{x}$ and the

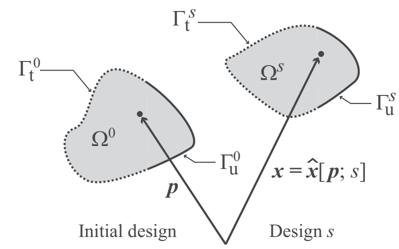

Fig. 1. Family of design domains Ω^s generated through design functions $\hat{\boldsymbol{x}}[\cdot; s]$.

meaning of the symbol may be inferred from the context (i.e., position or design mapping).

In the domain Ω^s and during a time interval $\mathcal{T} = [0, T]$, with T a given final analysis time, the governing equation for a transient heat conduction problem can be expressed as

$$l[\theta[\boldsymbol{x}, t]] := \rho c \frac{\partial \theta[\boldsymbol{x}, t]}{\partial t} - k\nabla^2\theta[\boldsymbol{x}, t] - Q[\boldsymbol{x}, t] = 0, \quad (\boldsymbol{x}, t) \in \Omega^s \times \mathcal{T}, \tag{1}$$

where $\theta = \theta[\boldsymbol{x}, t]$ is the temperature at point $\boldsymbol{x}$ and time $t, c > 0$ is the heat capacity, $\rho > 0$ is the mass density, $k > 0$ is the thermal conductivity, $Q = Q[\boldsymbol{x}, t]$ is the inner heat generation rate per unit volume (volumetric heat supply) and ∇^2 is the Laplacian operator. For subsequent use, the governing equation is written in terms of an operator l as defined in (1). It is assumed that there are only three types of boundary conditions on the boundary Γ^s, which is divided as follows:

$$\Gamma^s = \Gamma^s_\theta \cup \Gamma^s_{\mathrm{q}} \cup \Gamma^s_{\mathrm{e}}$$

where Γ^s_θ represents portions where the temperature is specified, Γ^s_{q} corresponds to portions where the contact heat supply is given and Γ^s_{e} is the part of the boundary where heat is exchanged with the environment through convection. The boundary and initial conditions are as follows:

$$\begin{aligned}
&\theta = \hat{\theta} \quad \text{on} \quad \Gamma^s_\theta \times \mathcal{T} \\
&\boldsymbol{q} \cdot \boldsymbol{n} = -\hat{q} \quad \text{on} \quad \Gamma^s_{\mathrm{q}} \times \mathcal{T} \\
&\boldsymbol{q} \cdot \boldsymbol{n} = -q_{\mathrm{e}} = h(\theta - \theta_{\mathrm{e}}) \quad \text{on} \quad \Gamma^s_{\mathrm{e}} \times \mathcal{T} \\
&\theta[\boldsymbol{x}, 0] = \theta_0[\boldsymbol{x}] \quad \text{in} \quad \Omega^s
\end{aligned} \tag{2}$$

with the heat flux vector $\boldsymbol{q}$ given by Fourier's model, i.e.,

$$\boldsymbol{q} = -k\nabla\theta.$$

In (2), $\hat{\theta} = \hat{\theta}[\boldsymbol{x}, t]$ and $\hat{q} = \hat{q}[\boldsymbol{x}, t]$ are the specified temperature and contact heat supply on $\Gamma^s_\theta \times \mathcal{T}$ and $\Gamma^s_{\mathrm{q}} \times \mathcal{T}$, respectively, $\boldsymbol{n} = \boldsymbol{n}[\boldsymbol{x}]$ is the unit outward normal vector to the boundary, h is the convection coefficient, $\theta_{\mathrm{e}} = \theta_{\mathrm{e}}[t]$ is the convective exchange temperature (ambient temperature) and $\theta_0[\boldsymbol{x}]$ is the initial temperature field in the domain. For notational convenience, the contact heat supply on Γ^s_{e} is denoted as q_{e}. The contact heat supplies $\hat{q}$ and q_{e} are given as the negative of the normal heat flux, hence they take positive values if heat flows from the external environment into the system and negative values otherwise.

2.2. Transient shape optimization problem

In order to develop a versatile framework for shape optimization problems, which can accommodate various situations within a unified formulation, a general objective functional is defined such that it can measure the performance on pre-selected parts where the physical phenomenon occurs, both in space and time. This is achieved through the combination of two local performance