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Attaching negative structures to model cut-outs in the vibration analysis of structures

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ABSTRACT

The presence of a hole, cut-out or void in a structure makes it difficult to be modelled for calculating natural frequencies. A theoretical basis for simplifying the modelling of cut-outs in a structure by attaching a negative structure is presented. The Dynamic Stiffness Method has been used to prove that this method yields the required natural frequencies. The derivations also show the presence of additional natural frequencies which correspond to the vibration of the positive and negative parts vibrating together while the actual structure with the hole or cut-out usually remains stationary.

1. Introduction

Determination of natural frequencies, critical loads and stress distribution in solid bodies with voids, holes, cut-outs or damages is increasingly becoming important in applications such as optimisation and damage detection [1–6]. The natural frequencies of such systems are commonly obtained using the Finite Element Method (FEM) [7–16]. There have also been some papers [17–19] which use analytical procedures such as the Rayleigh-Ritz Method (RRM) [20–22] in which the potential and kinetic energy terms of the structure are found by subtracting the energy terms associated with the void part from that of the larger structure (without the void), by taking the displacement forms of the void to be the same as those of the larger structure. In one recent approach [23], called the Independent Coordinate Coupling Method (ICCM), the displacement forms of the void and the larger domain are constrained to have similar values using Lagrangian type constraints in an average integral form. This method has been generalized in [24] which deals with the modelling of plate-like structures with holes as a basis for a structural optimisation process. Once the coupling is done, the energy terms corresponding to the void are subtracted from that of the larger structure and the Rayleigh-Ritz minimisation is then carried out.

This work stems from the authors' attempts to investigate the possibility of using the modes of both the structure without any void and a negative structure corresponding to the void and then combining the two sets of modes while enforcing the embedding continuity conditions by the penalty method [25,26] or the Lagrangian Multiplier Method (LMM) [27] in a Rayleigh-Ritz procedure. Numerical experimentation with this idea using discrete systems, beams and plates with cut-outs and holes showed that while it is possible to obtain the required frequencies, the presence of additional natural frequencies and the difficulty in choosing appropriate shape functions and constraint enforcement methods pose some challenges [28]. Thus the authors proceeded to study the theoretical basis for combining positive and negative structures, using the Dynamic Stiffness Method (DSM), the derivations and findings of which are presented in this paper, along with some numerical results. The paper shows that the required natural frequencies are obtainable from the model incorporating a negative structure, and explains the additional frequencies.

2. The theoretical basis

2.1. Existence of natural frequencies of the structure with a hole in the proposed model

In order to develop a theoretical framework, the question will first be addressed of whether or not all the required natural frequencies and modes of at least a certain class of structures

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containing voids can be obtained through the proposed method. The hypothesis taken is that these characteristics are obtainable by combining the modes of positive and negative structures, i.e. by embedding a negative structure with known modes into a larger positive structure with known modes, and analysing the combined system.

Consider the case of a structure A which contains a hole, represented in Fig. 1a by a rectangular plate with a circular hole. Now consider two elastic structural bodies C^+ and C^-, having identical shapes that would fill the void in A, one (C^+) with the same distribution of elastic and inertial properties as that of A, and the other (C^-) with negative properties but of the same magnitude. In this case C^+ and C^- will be circular plates. Then consider joining C^+ and C^- by means of a very stiff elastic continuum S_1, which acts as a penalty against any differential displacement between the two elements. The resulting structure has the potential to exist as an empty element (E). The term 'empty' is used here to indicate that the element could be subjected to any dynamic displacement without inducing any forces (or moments) at the boundaries. This empty element state occurs only in modes in which C^+ and C^- vibrate together with the same displacement. The combined unit, being empty, would not change the behaviour of any structure to which it is attached as there will not be any unbalanced forces or moments. This combined unit is now connected to Structure A along the common boundary with the hole (in this case having a circular circumference), by constraining the degrees of freedom to be the same. This may be done either by using a penalty parameter or by using sufficiently large number (say r) of discrete constraints. It may be seen that the resulting structure A′ (see Fig. 1b) is capable of possessing all the natural frequencies and modes of A, because it was formed just by the addition of the empty element E. Furthermore, if the penalty stiffness is sufficiently large, from the asymptotic modelling theorems [29], the combination of A and C^+ is equivalent to B (the rectangular plate without any hole). Connecting B to C^- using S_1 gives B′ as shown in Fig. 1c. It is therefore deduced that the natural frequencies and modes of B′ would include those of A. The hole in a plate is only an illustration but the same argument will hold for a body with cut-outs or voids. This will be proved formally for discrete systems in the next section.

2.2. Proof of existence of the required natural frequencies for discrete systems

Consider a discrete structural system A (Fig. 2a) having n_1 vibratory degrees of freedom which is obtainable from a larger system B (Fig. 2b) by removing some elements. A represents the structure with a cut-out and B is a larger structure which would be the result of filling the cut-out part. For simplicity, the discrete systems are represented with spring-mass arrangements. The proposed method involves attaching a negative structure C^- (Fig. 2c) to the larger positive structure B to obtain B′ as illustrated in Fig. 2d. Thus C^- would potentially cancel the stiffness and inertia in a part of A so as to produce A if it is rigidly connected to its positive counterpart within B. The masses associated with the degrees of freedom of A are shown as lightly filled circles and rectangles. The circles correspond to the internal degrees of freedom that do not lie on the common boundary with the hollow domain while the rectangles correspond to degrees of freedom that are on the boundary. The set of internal degrees of freedom of A will be denoted by vector $\mathbf{q_{Ai}}$ and the common degrees of freedom will be denoted by $\mathbf{q_e}$. Let the natural frequencies of A be $\boldsymbol{\omega_A}$, a vector set containing $\omega_{A,1}$, $\omega_{A,2}$, ..., $\omega_{A,n1}$. Now consider another discrete system C^+ (Fig. 2e) with m vibratory degrees of freedom which is the positive counterpart of C^- and represents the component of B if it did not have the cut-out. This means System C^- has the same magnitude of stiffness and inertial properties as C^+ but with opposite sign. The masses associated with the positive structure are shown as black circles or squares and their negative counterparts are shown as white circles and squares with dotted boundaries. Now consider linking these to system A at a common boundary where the masses associated with the shared degrees of freedom $\mathbf{q_e}$ are shown in squares. The circles depict the masses that are not associated with a common boundary with A (i.e. at internal degrees of freedom). These sets of internal degrees of freedom of C^+ and C^- are labelled $\mathbf{q_{Pi}}$ and $\mathbf{q_{Ni}}$ respectively, to indicate the internal degrees of freedom of the positive and negative structures. The natural frequencies of C^+ are $\boldsymbol{\omega_C} = [\omega_{C,1}, \omega_{C,2}, \ldots, \omega_{C,m}]$. As both inertial and elastic (stiffness) properties of C^- are equal and opposite to those of C^+, each term in the equation of motion of C^- would be equal and opposite to the corresponding term for C^+. This means the natural frequencies and modes of C^- are identical to those of C^+. For the purpose of this proof, it is necessary to consider System A′ shown in Fig. 2f, which is formed by joining A, C^+ and C^- rigidly at the common boundary (i.e. the degrees of freedom at the boundary between the three systems are common for them) and joining other corresponding degrees of freedom between C^+ and C^- by means of elastic springs S_1.

As the connection between A and C+ in A′ is rigid,

$$A' \equiv B' \tag{1}$$

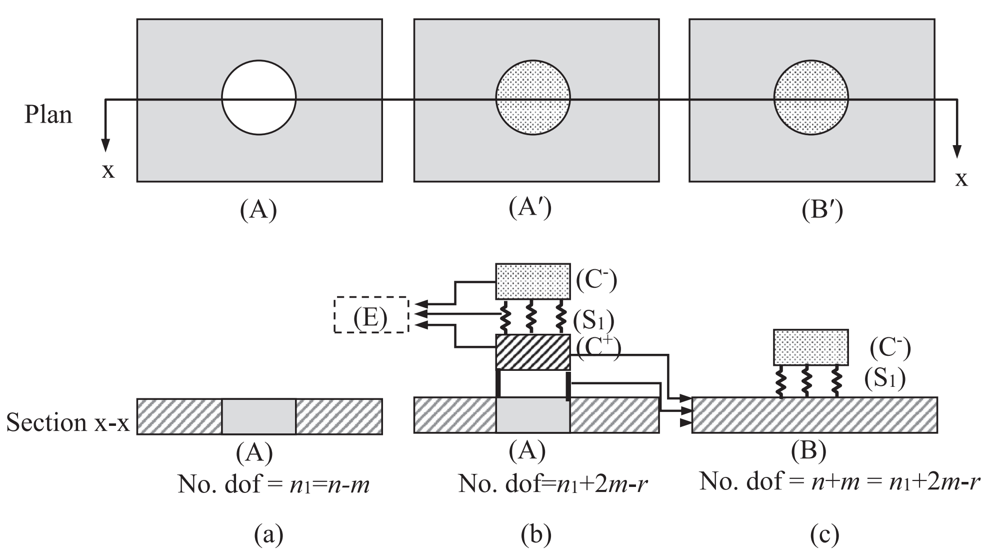

Fig. 1. Example of plates combining positive and negative structures.