Computers and Structures 184 (2017) 36–52

Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

Meshfree analysis of electromagnetic wave scattering from conducting targets: Formulation and computations

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ARTICLE INFO

Article history:
Received 13 November 2016
Accepted 27 January 2017
Available online 6 March 2017

Keywords:
Electromagnetics
Finite elements
Inf-sup condition
Meshfree methods
Mixed/hybrid formulations
Wave scattering

ABSTRACT

We propose a completely meshfree procedure aimed at the time-harmonic analysis of electromagnetic wave scattering from conducting targets. The problem is described by the vector wave equation with a divergence-free constraint. We propose a mixed formulation whose unknowns are the electric field vector and a Lagrange multiplier. We investigate the well-posedness of the variational problem and construct compatible meshfree function spaces able to describe solutions in any geometry, in two and three dimensions. The method does not depend on any kind of parameter tuning. We illustrate its performance in a number of solutions through experimentally derived convergence rates and comparisons with other techniques.

1. Introduction

Meshfree (or meshless) methods refer to a broad category of numerical procedures applied to the solution of differential equations. In some cases, the methods can be interpreted as a generalization of finite element methods [1–5]. The applications in research using meshfree methods are numerous, following the publication of some papers introducing these methods in computational mechanics, like the Smooth Particle Hydrodynamics (SPH) method [6], the Element Free Galerkin (EFG) method [7], the Local Boundary Integral Equation (LBIE) method [8,9], the Finite Spheres method [10,11], and the Meshless Local Petrov-Galerkin (MLPG) method [12].

Specific meshfree methods can be quite different from others. Features like the imposition of boundary conditions and the construction of interpolation functions, for example, are dealt with very differently depending on the method. However, there is one characteristic that is common to all meshfree methods: They rely on *nodes* scattered freely throughout the computational domain. There is no mesh or grid connecting these nodes. Indeed, it is one of the aims in the development of meshfree methods to circumvent the difficulties associated with the generation of a mesh, particularly in three dimensions. In some cases the methods construct independent basis functions defined on small regions around the nodes, called *subdomains*, *spheres*, or *patches*. These and generalizations thereof have recently been labeled ‘overlapping finite elements’ because the overlapping is the main characteristic distinguishing them from traditional finite elements [1,2]. The computational domain is covered by these overlapping elements. Comprehensive studies of meshfree methods in the solution of problems in mechanics led to the current research in applications of ever-increasing levels of complexity [13–21].

In electromagnetics, the introduction of meshfree methods as an alternative to the use of finite element methods came a few years later [22–25]. Some research in this field is focused on *collocation* procedures, i.e., methods which deal with the differential equations in *strong form*. They usually use Radial Point Interpolation (RPIM) basis functions, and can be seen as suitable alternatives to finite difference methods [26–28]. While simpler to implement, these methods suffer from instabilities or may not be fully meshfree [29].

Meshfree methods based on *weak forms* have also been considered. Some research has been focused on the EFG method [30–33], but because the EFG method relies on background cells to perform the numerical integrations, it is not considered a truly meshfree method. On the other hand the MLPG method is a truly meshfree procedure and has been used in electromagnetics, see for example [34–36].

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http://dx.doi.org/10.1016/j.compstruc.2017.01.014
0045-7949/

It is by now an established fact that meshfree methods based on weak forms can be used in electrical engineering, see also [37–39]. However, all the solution examples in [30–39] deal with *scalar field* problems. The extension of meshfree procedures based on variational forms to *vector field problems* in $\mathbb{R}^d$ ($d = 2$ or 3) is a significant step due to the difficulty in satisfying the divergence-free condition, and only few research efforts have been published, see for example [40]. Using finite element methods, vector field problems in electromagnetics are usually solved employing Nédélec edge elements [41–44]. In this approach, degrees of freedom are associated with each edge in the mesh (generally formed by, but not restricted to, triangles in 2D and tetrahedra in 3D), and the resulting basis functions are such that their divergence is zero within each element (but not at the element boundaries) [43].

In a true meshfree setting, we do not have the support of a mesh, which poses some difficulty in constructing appropriate vector basis functions. Some results using a meshfree procedure based on weak forms in the solution of vector electromagnetic field problems are for example given in [40]. The strategy in that work is to define vector basis functions on the patches. Despite the success, there are at least three points that deserve attention. First, the method has not been tested on curvilinear geometries. Second, the imposition of essential boundary conditions is based on Nitsche's method [45], in which the formulation incorporates to the weak forms extra regularization terms that depend on adjustable (tunable) stability parameters. Third, the vector basis functions defined on the patches must be subjected to an orthogonalization procedure in order to ensure that they are strongly linearly independent.

It is our aim to conceive a method that simply uses nodes scattered on the domain and scalar nodal basis functions. It does not resort to vector basis functions. The divergence-free condition is enforced weakly via a Lagrange multiplier that arises naturally when the double-curl operator in the vector wave equation is replaced by the vector Laplacian. We thus arrive at a system similar to the steady-state incompressible Navier-Stokes equations of hydrodynamics [46], for which there are reliable solution methods based on nodal finite elements [47,48]. The Lagrange multiplier p together with the scattered electric field $\boldsymbol{E}^s$ constitute the unknowns of the problem. However, in this mixed formulation the electric field and the Lagrange multiplier must reside in different function spaces. It is well-known that for a mixed formulation these spaces must be compatible via the *inf-sup* condition [48,49] (a fact used in the meshfree solution of a problem in mechanics in [50]). Also, since the conducting objects that scatter the incident wave can be of any shape, we show how to embed information about the shape of the scatterer into the meshfree spaces.

Considering the imposition of the essential boundary conditions, we impose these directly as in the standard finite element method, because our nodal basis functions satisfy the Kronecker delta property at the domain boundaries.

In the following sections we propose our method and then proceed to illustrate its application in several examples. Appendix A gives some discussion of the inf-sup condition which must be satisfied in order for the discretization to be reliable.

2. The differential equations of wave scattering

Let the conducting object be represented by a closed subset $\Sigma \subset \mathbb{R}^d$, as in Fig. 1. In this work, the canonical orthonormal basis for $\mathbb{R}^d$ is represented as $\{\hat{\boldsymbol{x}}_1, \dots, \hat{\boldsymbol{x}}_d\}$. The time-harmonic scattering of an electromagnetic wave by a perfect electric conductor (PEC) in free space is described in the differential formulation by the system of equations [43]:

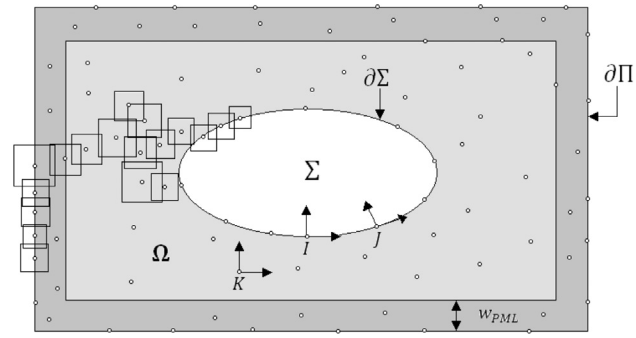

Fig. 1. The geometry of the scattering problem, illustrating the nodal distribution over the domain and along its boundary. (a) The computational domain Ω comprises the region between the outer contour $\partial\Pi$ and the surface $\partial\Sigma$ of the PEC object, which is represented by the white region (hole). (b) The PML corresponds to the layer adjacent to $\partial\Pi$. (c) The square patches, or 'overlapping elements' overlap each other (the collection of all overlapping elements associated with the nodes in the figure is not shown). (d) The patches do not conform to the geometry of the boundaries, as evidenced by the three patches at the PEC surface $\partial\Sigma$. (e) For the nodes I and J, located on $\partial\Sigma$, $\{\boldsymbol{\sigma}_1^I, \boldsymbol{\sigma}_2^I\}$ and $\{\boldsymbol{\sigma}_1^J, \boldsymbol{\sigma}_2^J\}$ are the normal and tangential unit vectors at their locations, whereas for the interior node K, $\{\boldsymbol{\sigma}_1^K, \boldsymbol{\sigma}_2^K\}$ are the unit vectors $\hat{\boldsymbol{x}}_1$ and $\hat{\boldsymbol{x}}_2$ along the coordinate axes X_1 and X_2, respectively. These ideas can naturally be extended to three-dimensional analysis.

$$\nabla \times \nabla \times \boldsymbol{E}^s - k_0^2 \boldsymbol{E}^s = 0, \quad in\ \Omega, \tag{1a}$$

$$\nabla \cdot \boldsymbol{E}^s = 0, \quad in\ \Omega, \tag{1b}$$

$$\hat{\boldsymbol{n}} \times \boldsymbol{E}^s = -\hat{\boldsymbol{n}} \times \boldsymbol{E}^{inc.}, \quad on\ \partial\Omega, \tag{1c}$$

$$\lim_{r\to\infty} \hat{\boldsymbol{r}} \times \nabla \times \boldsymbol{E}^s = jk_0 \boldsymbol{E}^s. \tag{1d}$$

The function $\boldsymbol{E}^s : \Omega \to \mathbb{C}^d$ is a *phasor*; once it has been calculated, the real scattered electric field is given by $\boldsymbol{\mathcal{E}}^s = \mathrm{Re}\{\boldsymbol{E}^s e^{j\omega t}\}$, where $\omega = 2\pi f$ (f is the wave frequency), Re denotes the real part of a complex number, $j = \sqrt{-1}$, and t represents time. The scattering problem is stated in the unbounded domain (see Fig. 1):

$$\Omega = \mathbb{R}^d - \Sigma, \tag{1e}$$

with the boundary:

$$\partial\Omega = \partial\Sigma. \tag{1f}$$

In (1a) and (1d), $k_0 = \omega\sqrt{\mu_0 \varepsilon_0}$ is the *propagation constant* in free space; μ_0 and ε_0 are the free-space magnetic permeability and electric permittivity, respectively. Since there are no losses, k_0 is a real number. On the surface of the PEC object, the boundary conditions are given by $\hat{\boldsymbol{n}} \times \boldsymbol{E} = 0$, where $\hat{\boldsymbol{n}}$ is an outward-pointing unit normal vector at the surface $\partial\Omega$ of the domain (and which points towards the interior of Σ). The total electric field is $\boldsymbol{E} = \boldsymbol{E}^s + \boldsymbol{E}^{inc.}$, given by the sum of the scattered and incident fields, where the incident field is prescribed [51]. The expression in (1d) is the *radiation boundary condition*, where $\hat{\boldsymbol{r}}$ is the unit vector in the direction of the radius vector $\boldsymbol{r}$ (from the origin $\mathcal{O}$ to any point of $\mathbb{R}^3$), and r is the Euclidean norm of $\boldsymbol{r}$, i.e., $\boldsymbol{r} = \|\boldsymbol{r}\|\hat{\boldsymbol{r}} = r\hat{\boldsymbol{r}}$. This condition ensures that the scattered field $\boldsymbol{E}^s$ propagates away from the PEC object [43].

The electric field in (1a) is constrained by the condition (1b), that is, Gauss' law for the free-space with no sources. To develop our formulation, we use the vector identity $\nabla \times \nabla \times \boldsymbol{E}^s = -\nabla^2 \boldsymbol{E}^s + \nabla(\nabla \cdot \boldsymbol{E}^s)$, so that (1a) becomes:

$$\nabla^2 \boldsymbol{E}^s + k_0^2 \boldsymbol{E}^s - \nabla(\nabla \cdot \boldsymbol{E}^s) = 0, \quad in\ \Omega. \tag{2}$$