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Stochastic optimization of regulators

Kurt Marti

Federal Armed Forces University Munich, Aerospace Engineering and Technology, 85577 Neubiberg, Munich, Germany

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ABSTRACT

The optimal design of regulators is often based on the use of given, fixed nominal values of initial conditions, external loads and dynamic parameters of the control system. However, due to variations of material properties, tasks to be executed, modeling errors, etc., the model parameters are not exactly known and given quantities. In addition, the state of the system cannot be observed or measured exactly there are always some observational/measurement errors. Thus, a predetermined (optimal) regulator should be robust, i.e., the regulator should guarantee satisfying results also in case of observational errors and errors in the selection of the initial conditions, external load parameters, dynamic parameters, etc. Since uncertainties can be modeled and recorded very efficiently by probabilistic terms, in contrast to other approaches in optimal regulator design, the occurring errors are modeled here by realizations of random variables having a given or at least partly known probability distribution. Thus, instead of calculating optimal regulators by solving very complex minimax optimization problems, here, robust optimal regulators can be found by means of stochastic optimization methods. Using Taylor expansion methods for calculating the occurring expectations, stochastic optimal regulators can be determined by deterministic optimization problems which can be handled partly analytically for additive as well as multiplicative measurement errors. Moreover, the procedure indicates how the gain matrices must be selected in order to get stable perturbation equations for the sensitivities. Finally, the given procedure is applied to the important field of *active control of mechanical structures*. An illustrative example is given.

1. Introduction

The optimal design of regulators is often based on the use of given, fixed nominal values of initial conditions, load and other model parameters. However, due to *variations of the material properties, measurement errors (e.g. in case of parameter identification), modeling errors (complexity of real systems), uncertainty on the working environment, the task to be executed, etc.*, the true initial conditions, external load and further model parameters, like sizing parameters, mass values, gravity centers, moments of inertia, friction, tolerances, adjustment setting error, etc., are not known exactly in practice. Hence, a predetermined (optimal) regulator should be "*robust*", i.e., the controller should guarantee satisfying results also in case of variations of the initial conditions, load and other model parameters.

Robust controls have been considered up to now mainly for uncertainty models based on given fixed sets of parameters, like multiple intervals, assumed to contain the unknown, true parameter. In this case one requires then often that the controlled system fulfills certain properties, as e.g. certain stability properties for all parameter vectors in the given parameter domain. If the required property can be described by a scalar criterion, then the controller design is based on a minimax criterion, such as the H^{∞}-criterion, see e.g. [1,4,9,10].

Since in many cases parameter uncertainty can be modeled more adequately by means of stochastic parameter models, in the following we suppose that the parameters involved in the regulator design problem are realizations of a random vector having a known or at least partly known joint probability distribution. The determination of an optimal controller under uncertainty with respect to varying material properties, manufacturing procedures, working neighborhoods, modeling assumptions, etc., is a *decision theoretical problem*. Criteria of the type "*holds for all parameter vectors in a given set*" and the *minmax*-criterion are very pessimistic and often too strong. Indeed, in many cases the available a priori and empirical information about the dynamic system and its working neighborhood allows a more adequate, flexible description of the uncertainty situation by means of stochastic approaches. Thus, it is often more appropriate to model unknown and varying initial values, external loads, measurement errors and other model parameters as well as modeling errors, e.g. incomplete representation of the dynamic system, by means of realizations of a random

E-mail address: kurt.marti@unibw-muenchen.de
URL: http://www.stoch.net

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vector, a random function with a given or at least partly known probability distribution. Consequently, the optimal design of robust regulators is based on an optimization problem under stochastic uncertainty [2,16,26].

1.1. Stochastic optimal design of regulator

For the consideration of stochastic parameter variations as well as other uncertainties within the optimal design process of a regulator one has to introduce – as for any other optimization problem under (stochastic) uncertainty – an appropriate **deterministic substitute problem**. In the present case of stochastic optimal design of a regulator, hence, a map from the state or observation space into the space of control corrections, one has a **control problem under stochastic uncertainty**. Then, for the solution of the occurring deterministic substitute problems the methods of *stochastic optimization*, cf. [16,26], are available.

As is well known [3,19,20], the optimization of a regulator presupposes an optimal reference trajectory $q^R(t)$ and a corresponding feedforward control $u^R(t)$. In case of stochastic uncertainties, the guiding functions $(q^R(t), u^R(t))$ can be determined also by stochastic optimization methods [3,14,21].

The computation of stochastic optimal regulators is based now on deterministic substitute control problems of the following type:

Minimize the expected total costs composed of (i) the costs arising from the deviation $\Delta z(t)$ between the (stochastic optimal) reference trajectory and the effective trajectory of the dynamic system and (ii) the costs for the control corrections $\Delta u(t)$

subject to the following constraints:

- *dynamic equation of the underlying stochastic system with the total control input $u(t) = u^R(t) + \Delta u(t)$ being the sum of the fedforward control $u^R(t)$ and the control correction $\Delta u(t) = \varphi(t, \Delta z(t))$,*
- *stochastic initial conditions $q(t_0) = q_0(\omega), \dot{q}(t_0) = \dot{q}_0(\omega)$ for the state of the system at the starting time point t_0,*
- *conditions for the feedback law $\varphi = \varphi(t, \Delta z(t))$, such as $\varphi(t, 0) = 0$ (if the effective state is equal to the state prescribed by the reference trajectory, then no control correction is needed).*

Here, as often in practice, we use quadratic cost functions. The resulting deterministic substitute problem can be interpreted again as a control problem for the unknown feedback control law $\Delta u(t) = \varphi(t, \Delta z(t))$. A main problem is then the computation of the (conditional) expectation arising in the objective function. Since the expectations are defined by multiple integrals, the expectations can be determined in general only approximatively. In the following, approximations based on Taylor expansions with respect to the stochastic parameter vector $a(\omega)$ at its conditional mean $\bar{a}$ are taken into account. Using quadratic cost functions and first order Taylor expansions, hence, linearizations, one has the advantage that often a certain part of the calculations can be done analytically!

1.2. An important application: active control under stochastic uncertainty

In Section 6 the present method is applied to the very important area of "*active control under stochastic uncertainty*": In order to stabilize mechanical structures under strong dynamic applied loads, active control strategies are taken into account [5,22,23]. Without heavy external dynamic disturbances, such as strong earthquakes, wind turbulences, water waves, etc., mechanical structures usually are stationary, safe and stable. However, in case of heavy dynamic disturbances, additional control elements can be installed enabling active control actions. Active control strategies for mechanical structures are applied then in order to counteract heavy applied dynamic loads, which would lead to large vibrations causing possible damages of the structure. Describing the structural dynamics by means of a system of first order differential equations with random parameters for the state vector (displacement vector q and the time derivative of q), robust optimal controls are determined in order to cope with the stochastic uncertainty involved in the dynamic parameters, the initial values and the applied loadings. The problem is modeled in the framework of optimal control for minimizing the expected total costs arising from the tracking error (deviation from the reference trajectory) of the structure and the regulation costs. A numerical example is given.

Remark 1.1. The present paper is an updated and extended version of the conference paper [15]. Here, a new Section 1 "Introduction" has been included presenting the state of the art in this field and the aim of the paper, as e.g. a description of the advantages of the stochastic optimal regulator design – in comparison with other regulator optimization methods in case of uncertainty. The stochastic optimization method for the optimal regulator design under stochastic uncertainty, as developed in the paper, has been applied in a new Section 6 to the area of "Active Structural Control under Stochastic Uncertainty". Hence, an important practical application from structural design, which demonstrates the functioning of the proposed stochastic optimization method, has been included on the one hand, and an extension of the conference paper into the direction of engineering applications has been provided on the other hand. Active Structural Control is an important tool used in optimal structural design to stabilize structures, like large buildings, by mounting active control units to counteract large external loads from strong earthquakes, wind turbulences, water waves, etc. Here, besides randomly varying structural parameters, also the external loads have an uncertain character which can be described favorably by means of probabilistic models. A numerical example is given. In addition, the introduction of this Section 6 describes the reason and aims of active/semi-active structural control, where appropriate references on "active control" are provided: [5,12,18,22–25,27].

Moreover, the following additional improvements and amendments have been provided: (a) In the introduction other, known possible approaches [1,4,9,10] for regulator optimization under uncertainty are discussed and compared with the present one. (b) Then, a result concerning the stability property of the stochastic optimal closed loop system with a PID-regulator has been included, see Lemma 4.1. (c) In Chapter 5 (former Chapter 4), a more explicit representation of the final deterministic control problem is given. Especially, the system of integro-differential equations has been converted into a first order differential equation, see (52d). (d) In the present revision also more details about the modeling and treatment of measurement/observational errors are given: The definition of the additive and multiplicative stochastic measurement/observational errors has been improved. Furthermore, a more detailed analysis of the influence of measurement errors, see (6a)–(6i), has been given for the case of (i) additive as well as for the case of (ii) additive and multiplicative errors.

2. Regulator design under stochastic uncertainty

Feedforward and feedback control is based on the dynamic equation of the underlying control system

$$F(p_D, q(t), \dot{q}(t), \ddot{q}(t)) = u(t), \quad t \geqslant t_0 \tag{1a}$$

$$q(t_0) = q_0, \quad \dot{q}(t_0) = \dot{q}_0. \tag{1b}$$