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A model for heat transfer in cohesive cracks

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ABSTRACT

This paper is concerned with modeling of heat flow through cracks in three-dimensional thermo-mechanical problems. Various aspects connected with heat flow through crack are analyzed in the model. The analysis leads to the model that takes into account the following elements: crack opening, crack sliding, conductivity through bridging elements, conductivity through air between crack faces and heat radiation between crack faces. The model for the crack heat flow is combined with cohesive crack model. The global formulation is evaluated for the problem which is nonlinear due to nonlinear relations in the applied crack model, so the linearization is proposed. Both of the fields, namely the displacement and the temperature field, are discontinuous along the crack surface. Therefore, the solution procedure is based on XFEM approach that is applied for both mechanical and thermal parts. In consequence, the coupled linearized system of equations is evaluated.

1. Introduction

The paper presents an analysis of a combined thermo-mechanical problem in a fractured structure. Some kinds of engineering structures have to be analyzed with respect to their possible failure in the case of combined mechanical load and fire exposure. The exposure of a material to a high temperature due to a fire in its working environment can lead to destructive results which may affect the structure. Under high temperatures the material properties can drastically change (e.g. [1,2] for concrete). As shown in many experimental investigations, long term exposition to high temperatures irreversibly changes mechanical features of a material. The paper presents the analysis of heat flow through a crack in three-dimensional thermo-elastic domain. Many engineering structures, besides mechanical loading, are subjected to thermal loading. The thermal loading may be intentional or unintentional in case of fire, for instance. The behavior of a material in a situation of severe thermal loading needs to be predictable in practical engineering. In fact, many engineering structures have to be designed to withstand fires.

The cohesive crack model is used in the paper which has been applied by many researchers for a vast range of problems connected with crack growth (e.g. [3–7] and many others). The application of the cohesive crack model to combined thermo-mechanical problems has also remained in the scope of interest of some researchers (e.g. [8–12]). The fracture in the material introduces the anisotropy that affects both mechanical and thermal fields. The opened crack acts as a natural barrier for the heat free flow through the material. The topic has been investigated by many researchers, e.g. [13–16]. The results presented in these papers acknowledge that heat flux through crack is different than through continuous material, and this leads to temperature jump in the material along the crack.

The paper deals with three-dimensional (3D) analysis of thermo-mechanical cohesive crack model, where the cohesive tractions and the heat transport through the crack are taken into account. All calculations associated with the crack are performed in a two-dimensional (2D) subspace since the crack opening vector is decomposed into normal and sliding parts. Such decomposition was first proposed for explicit 3D cohesive crack analysis in [17]. The idea of decomposing the crack opening vector in this way has subsequently been applied successfully in a number of other publications, for example in [18–22]. In an other paper [23], the idea of the vector decomposition has been pursued to efficiently obtain tangent stiffness matrix for such kind of problems. The discontinuity in approximation fields can be introduced by doubling nodes in the finite element mesh, as [24] for example. However, nowadays the most popular way for that is using the XFEM approach and level set method for defining the crack geometry, e.g. in [25–29] and many others. In this paper, the XFEM approach has been combined with the level set method. The XFEM allows to define the crack geometry independently of the finite element mesh.

The heat transport across the opened crack depends on many factors. It is rather difficult to apply a function taking into account

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all of the elements. In the article [30] the crack heat flow was modeled as a combination of conduction and radiation. Another paper [11], presents the crack heat conduction through bridging fibers and through air between crack faces. In [9], the thermo-mechanical viscoelastic cohesive model was developed in large deformations, while in [12] the same kind of problem is applied for debonding in bimaterials. We can take an advantage, here, from the fluid flow through the crack in porous media presented e.g. in [31–33]. There are quite a lot in common in the physical description of heat and fluid flows through cracked media. In both cases the fluxes over the cracks appear and their values depend on the jumps of temperature or velocity for heat or fluid, respectively.

In this work we consider crack heat flux in three-dimensional model that is combined with growing cohesive crack model. Various aspects connected with heat flow through crack are analyzed in the model. The analysis presented here produces model that takes into account the following elements: (i) crack opening, (ii) crack sliding, (iii) conductivity through bridging elements, (iv) conductivity through air between crack faces, (v) heat radiation between crack faces. It is supposed that the material is heated up to high levels of temperature. As stated above, material parameters at high temperatures, such as Young modulus or thermal conductivity, can be changed radically. The same situation can occur for other parameters, such as tensile strength, fracture energy and others. In consequence, temperature affects the parameters related to cohesive zone model. This leads to the situation where crack cohesive forces can be changed due to temperature change at the crack surface. The change of cohesive tractions causes the feedback because crack opening directly depends on cohesive tractions, while heat flux through the crack is a function of the crack opening. Finally, a highly nonlinear problem arises that has to be solved.

The paper is based upon Author [34,35], and it combines the both topics in one paper, it also includes additional examples. The paper is the Author's direct continuation of the research presented in [10,36].

First in Section 2, the mathematical problem is formulated in detail whereas the mechanical (2.1) and thermal (2.2) mathematical models are shown in global formulations. In Section 2, some nonlinear relations related to thermo-mechanical cohesive model are introduced. Their linearization is shown in Section 3, which ends with linearized global formulations of mathematical model of the problem. In the next Section 4, the model for crack heat flow is developed. The Section is divided into parts related to conduction, radiation and their combination and ends in a presentation of the final model of heat transport through the crack. The Section also deals with dependence of cohesive law on current temperature. The important element of the analysis is approximation of displacement and temperature fields. Both of the fields are discontinuous so their effective approximation is done by XFEM type approximation for both displacement and temperature fields, which is shown in Section 5. The mathematical model requires values of jumps between two sides of the crack and also the mean value of values on both sides of the crack. By using the XFEM shape functions, the requirements can be quite easily fulfilled. Section 5 ends with linearized system of equations. The model developed in this paper is illustrated with three-dimensional examples that present the effectiveness of the proposed approach. The final example presents the non-stationary crack propagation in thermo-mechanical domain. The paper ends with some conclusions in Section 8.

2. Mathematical model of a problem

The computational model in the paper is derived for fully three-dimensional case assuming small strains and displacements. The domain under consideration V with outer boundary S is discontinuous at S_d crack surface. The domain, outer boundary, crack and the crack two-dimensional local coordinates defined on unit vectors $(\mathbf{n}_d, \mathbf{s}_d)$ are shown in Fig. 1. All the equations related to the crack are described in the crack two-dimensional local coordinates. The crack normal $\mathbf{n}_d$ comes from the geometry of the crack and is known in advance. The second unit vector $\mathbf{s}_d$ shows the direction of the sliding part of the crack opening vector and has to be established in the calculations. The existence of the crack in the domain, in the coupled thermo-mechanical problem, leads to discontinuities in displacements and in temperature fields.

2.1. Mathematical model of mechanical part

The analysis begins with the standard equilibrium equation (momentum balance) which is valid at each point of the considered isotropic solid V and for each moment of time t

$$\operatorname{div}\boldsymbol{\sigma} + \mathbf{b} = \mathbf{0} \qquad \text{in } V \tag{1}$$

where $\boldsymbol{\sigma}$ is the stress tensor and $\mathbf{b}$ is the body force vector. The equation is completed by the following standard boundary conditions

$$\boldsymbol{\sigma}\mathbf{n} = \mathbf{t} \qquad \text{on } S_\sigma, \qquad \mathbf{u} = \hat{\mathbf{u}} \qquad \text{on } S_u \tag{2}$$

where $\mathbf{t}$ is a traction forces vector, $\mathbf{u}$ is a displacement vector and $\hat{\mathbf{u}}$ is prescribed displacement vector. The crack existing inside the body is described by surface S_d. Between both sides of the crack surface the discontinuity in the displacement vector $\mathbf{u}$ is observed and on both sides of the crack surface the cohesive forces are applied which can be given in the form similar to natural boundary condition

$$\boldsymbol{\sigma}\mathbf{n}_d = \mathbf{t}_d \qquad \text{on } S_d \tag{3}$$

where $\mathbf{n}_d$ is the unit normal vector to S_d, and $\mathbf{t}_d$ is the cohesive tractions vector.

The mathematical model from Eqs. (1)–(3) is reworked into a global weak formulation using the weighted residuum approach, using test function $\mathbf{v}_u$. Both of the functions $\mathbf{u}$ and $\mathbf{v}_u$ are discontinuous along S_d. Therefore, Dirac's delta appears in S_d while differentiating these fields. When the gradient operator is applied to test function $\mathbf{v}_u$ which is discontinuous on S_d, it results in

$$\nabla(\mathbf{v}_u) = \begin{cases} \nabla\mathbf{v}_u & \text{for} \quad \mathbf{x} \notin S_d \\ \delta_{S_d}[[\mathbf{v}_u]] \otimes \mathbf{n}_d & \text{for} \quad \mathbf{x} \in S_d \end{cases} \tag{4}$$

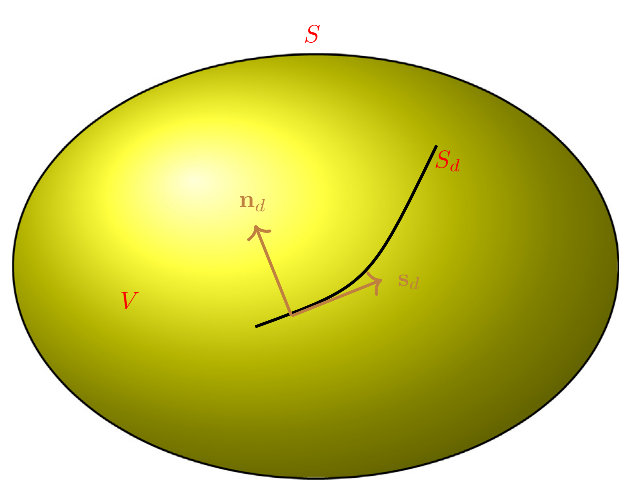

Fig. 1. Domain with inner crack and crack local coordinates.