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Buckling of thin-walled structures through a higher order beam model

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ABSTRACT

A higher order beam model for the buckling analysis of thin-walled structures is presented in this paper. The model considers an enrichment of the displacement field so as to accurately represent the three-dimensional behaviour of thin-walled structures. The definition of an uncoupled set of deformation modes allows a meaningful definition of hierarchical higher order solutions, which are useful for the linear buckling analysis of thin-walled structures. A criterion for the definition of local and global buckling modes, as well as possible interaction between modes is put forward. A comparison between the results obtained with the higher order beam model and results obtained from a shell finite element model implemented in Abaqus allows to conclude not only the efficiency of the beam model but also its simplicity of use.

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1. Introduction

Thin-walled structures are prone to both local and global buckling phenomena, being the design of this type of structures often determined by the interaction between theses modes. The stability analysis by a shell finite element model is an adequate procedure to evaluate bifurcation loads and the corresponding buckling modes. Nonetheless, shell finite models, although being accurate in modelling the 3D structural behaviour of thin-walled structures, can pose certain difficulties in the interpretation of buckling phenomena, particularly when interaction between modes of different wavelengths occurs.

Alternatively to shell models, beam models can also be efficiently adopted to perform a stability analysis of a thin-walled structure, having the advantage of allowing a more clear insight into the buckling phenomena. To this end, the beam model has to be capable of reproducing the 3D structural behaviour of the thin-walled structure, namely the corresponding out of plane warping and the in-plane flexure of a cross-section. Moreover, the beam model should be able of providing sets of uncoupled solutions for the stability of the thin-walled structure so as to clearly identify the most relevant buckling modes.

Although considering different approaches, several beam models have been successfully adopted to model thin-walled structures. Essentially, these one-dimensional models rely on the enrichment of the displacement field over the cross-section in order to enhance the accuracy of the model. The quantification of the Saint–Venant principle has been adopted in the formulation of beam models by defining the three-dimensional continuum mechanics in terms of higher order modes [30,15,13,14,16–18]. The displacement field of beam models has also been enhanced by adopting (i) an asymptotical analysis of the cross-section [21,48]; (ii) an approximation through a Taylor's expansion [8–10] and (iii) an approximation of the displacement field on the beam cross-section by a set of linearly independent basis functions [37,33,38,34,35,25,24,26–28,32,41,12]. The so-called generalised beam theory (GBT), which has been developed from the seminal work of [43] towards its applicability to more generic cross-section midline geometries [11,19,20,36,23,22,39,29], is a successful theory for the analysis of thin-walled structures; the GBT owes its success to its modal uncoupled nature, which renders the theory an adequate tool for the buckling analysis of thin-walled structures. Thin-walled structures have also been analysed through a semi-analytical finite strip analysis (the constrained finite strip analysis – cFSM), being the corresponding mechanical behaviour evaluated through the separation of the corresponding deformation modes [42,4,3,2]. A comparison between the modal approaches of GBT and cFSM has been presented in [5].

The higher order beam model that considers the out of plane warping and the in-plane flexure of thin-walled structures presented in [44,45,47] has been applied to the buckling analysis of thin-walled structures with closed hollow sections in [46], being herein applied to an I shaped cross-section, allowing to prove the versatility of the model in dealing with different cross-section geometries.

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The model copes with the loss of accuracy inherent from the reduction of a three dimensional elasticity formulation to a one-dimensional model by an enrichment of the beam displacement field on the beam cross-section through a set of interpolation functions with a suitable degree, defined over a sufficiently refined mesh of the cross-section. The beam governing equations are derived considering the approximation scheme adopted for the displacement, yielding a set of fourth order differential system of equations. The solution of this system is analysed through the corresponding non-linear eigenvalue problem, which allows to obtain a set of uncoupled deformation modes.

The proposed model, in what regards the enrichment of the displacement field approximation, has a pure kinematically unconstrained approach in as much as it considers an interpolation of the displacement components by a set of linear independent functions defined element-wise on the beam cross-section domain.

On the other hand, the so-called conventional GBT formulation, developed to analyse thin-walled structures with an open and unbranched midline profile, considers the null membrane shear hypothesis on the definition of the displacement field; moreover, it also considers the in-plane components of the displacement field defined by analysing the cross-section as a plane frame subjected to imposed displacements. To consider other types of cross-sections, as well as additional deformation modes, several extensions/enhancements of GBT have been successfully adopted [11,19,20]. However, these new GBT formulations remain considering the cross-section mechanical behaviour as a procedure for the approximation of the displacement field, e.g., by imposing unit displacements at specific nodes whilst enforcing others displacements to be null and in compliance with deformation assumptions or by imposing unit deformations at each wall while keeping the deformations of other walls null. Similar mechanical assumptions are also considered in the cFSM formulation so as to constrain the corresponding equilibrium equations to a set of deformations according to a certain criteria. Conversely, the proposed higher order thin-walled beam model has a different approach regarding the procedure adopted for the approximation of the displacement field. In fact, by considering the definition of the displacement field independent of the cross-section mechanical behaviour, the proposed model can successfully be applied to any cross-section type, allowing to consider all cross-section membrane deformation components. Moreover, classic and higher order deformation modes (either in-plane and out-of-plane deformation modes) are obtained without having to enforce hypotheses on the structural behaviour, e.g., the Vlassov's assumption of null membrane shear is not *ab initio* considered, being nevertheless the classic and higher order warping modes of thin-walled structures obtained.

Towards the successful application of the higher order beam theory, a concept for the definition of uncoupled deformation modes has been recently put forward by the authors [45]. This concept relies on the solution of a non-linear eigenvalue problem associated with the governing equations of the model; a twelve fold null eigenvalue is obtained, which through a computation of a Jordan chain of matrices allows to retrieve classic solutions as well as a set of non-null eigenvalues that define higher order solutions hierarchically sorted. As a criterion to evaluate the accuracy of the uncoupling procedure, the eigenvalues of the governing equations written in the new orthogonal coordinates should render exactly the same set of eigenvalues. By rewriting the governing equations for a linear stability analysis and adopting as coordinates the orthogonal displacement modes, the obtained system of equations is not fully uncoupled. In fact, the remaining coupled terms allow a meaningful definition of the thin-walled structural phenomena.

The buckling modes are then defined as linear combinations of orthogonal displacement modes, which are defined according to the coupled terms of the linear stability governing equations rewritten in the new orthogonal coordinates. GBT formulations consider an uncoupling strategy that sought the diagonalization of the linear stability governing equations to the maximum extent form through the solution of a sequential set of linear eigenvalue problems between the corresponding coefficient matrices, being therefore the geometric coefficient matrix also considered in the process. The cFSM model approach considers a linear eigenvalue problem between the elastic stiffness and the geometric stiffness matrices obtained for a finite strip discretisation of the thin-walled structure [3,2]. These matrices are obtained considering a set of mechanical assumptions so as to constrain the thin-walled structure to deform according to certain criteria.

2. A higher order thin-walled beam model

A beam model derived by considering an enriched approximation of the displacement field so as to accurately represent the 3D structural behaviour of a thin-walled structure is considered. The model relies on the approximation of the displacement field on the cross-section by a set of linearly independent basis functions and on a criterion to uncouple the corresponding governing equations for an efficient analysis of thin-walled structures.

The cross-section is divided into elements for the approximation of the displacement field, being each displacement component interpolated independently along each element.

To this end, the formulation considers a cross-section represented by a set of rectilinear wall segments, which form a generic shape, either of open or closed profile. A rectilinear segment can be divided into several elements for better approximation features. Each element is considered to have both a flexural and a membrane structural behaviour. For the flexural behaviour, it is assumed that the elements are sufficiently "thin" in order to consider valid the *Kirchhoff* formulation and hence neglect the shear deformation on the planes perpendicular to the middle surface. The displacement field considering both the membrane and the flexural structural behaviour of the plate is defined as follows:

$$u_x(x,s,n) = \bar{u}_x(x,s) - n\,\frac{\partial u_n}{\partial x}, \quad u_s(x,s,n) = \bar{u}_s(x,s) - n\,\frac{\partial u_n}{\partial s} \quad \text{and} \quad u_n(x,s,n) = \bar{u}_n(x,s) \tag{1}$$

where x represents the beam axis; s the coordinate along the cross-section profile and n the corresponding perpendicular; $u_x(x,s)$ and $u_s(x,s)$ represent the in-plane displacements associated with the plate membrane behaviour and $u_n(x,s,n)$, the plate transverse displacement, which is considered to be constant over the plate element thickness.

The displacement field of the cross-section middle surface is thus approximated independently in the three spatial directions through the following expressions,

$$\bar{u}_x(x,s) = \boldsymbol{\phi}^t\,\mathbf{u}_x, \quad \bar{u}_s(x,s) = \boldsymbol{\psi}^t\,\mathbf{u}_s \quad \text{and} \quad \bar{u}_n(x,s) = \boldsymbol{\chi}^t\,\mathbf{u}_n \tag{2}$$

where $\boldsymbol{\phi}$, $\boldsymbol{\psi}$ and $\boldsymbol{\chi}$ correspond to the sets of the adopted basis functions; the basis functions are defined by assembling the approximation of the displacement field over each thin-walled element. The presented model was derived considering the approximation of the displacement field over the beam cross-section through a set of globally defined basis functions. However, it is more versatile, and computationally more efficient, to divide the cross-section into a set of rectilinear laminar elements, forming a cross-section mesh, and to consider an approximation for the displacement field defined over each thin-walled element.

The assemblage of the cross-section "elements" is made by considering the compatibility in terms of the middle surface

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