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Innovative straight formulation for plate in bending

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ABSTRACT

In this paper it has been introduced an innovative formulation for evaluating the deflection function of a simply supported plate loaded by uniformly distributed edge moments. Framed into Line Element-less Method, this formulation allows the evaluation of solution in terms of deflection, through few lines of algorithm implemented by Mathematica software without resorting to any discretization neither in the domain nor in the boundary. Interesting savings in terms of time and computational costs are achieved. Results obtained by the proposed method are well contrasted by ones obtained by classical methods and Finite Element Method.

1. Introduction

Capturing the structural behavior through solving the formulation of physical problems, is the constant interest of engineer and scientist researches working on civil, mechanical construction field. Generally the equations governing the problem are partial differential equations (PDEs) and, since exact solutions are available just for a restricted range of application, engineers and scientists in the second past of the last century started to improve numerical procedures. The main idea of numerical simulation is to transform a complex practical problem into a simple discrete form of mathematical formulation representing the problem of concern. The method used for numerical analysis of structures during the last 30 years is mainly the Finite Element Method (FEM) [1]. Then the Boundary Element Method (BEM) [2–7] was an alternative tool for numerical analysis, but in late Meshless or Meshfree (MFree) method has been developed with a great success [8,9]. Basically FEM needs a discretization over the entire domain through finite element mesh. Modification of the discretized model to improve the accuracy of the solution may be cumbersome. Although FEM evaluates the field function accurately, it is not proper to determine its (higher order) derivatives. BEM overcomes these latter drawbacks since the discretization is only over the boundary of the body, thus to remodel will be very easy. Moreover the BEM allows evaluation of the solution and its derivatives at any point of the domain! It is apparent that the challenge for improving these numerical methods is getting rid of the elements and meshes and, based on that, the meshfree (MFree) or Meshless methods have been developed.

The definition of Meshfree method [8] is: Meshfree method is a method used to establish system algebraic equations for the whole problem domain without the use of a predefined mesh for the domain discretization.

Recently, a truly no-mesh method has been proposed for solving beam under torsion [10–14]. Such a method, called Line Element-less Method (LEM) does not need any discretization, all integrals are simple line integral even those used for evaluating the properties of cross-section as area, moment of inertia [15].

Based on the analogy between the bending of simply supported plates loaded by uniformly distributed edge moments and the torsion of a beam [16–18], LEM for torsion problems has been used for solving plate in bending [19].

This paper deals with direct method for solution of thin plate bending problems when the plate is subjected to constant bending moment along whole simply supported edge. Then, the plate deflection is governed by the Poisson equation with constant right hand side. The solution for deflection is given by superposition of the particular solution and the harmonic polynomials obeying the Laplace equation. The expansion coefficients are determined by satisfaction of prescribed boundary conditions (b.c.) on the boundary edge of the plate (vanishing deflections, since the rest of b.c. given by constant bending moment is satisfied).

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Such a method with respect to the well-known Trefftz methods [20], where b.c. are enforced at the $N(=2n-1)$ boundary points in order to calculate N expansion coefficients, is element-free. The b.c. are satisfied in least square sense on the boundary edge. Then, it is necessary to perform only line integrations along the parametric equation of boundary edge in order to get the system of algebraic equations for the unknown expansion coefficients.

This innovative formulation, proper for evaluating the deflection function of a simply supported plate loaded by uniformly distributed edge moments, implemented through few lines of Mathematica software leads to the exact solution whenever it exists, or to a reliable solution for all cases and thus LEM may be also applied for plates having any shape, getting rid from the analogy with twisted beam.

Numerical results, which show the elegance and efficiency of the method will be reported contrasted with results obtained by using a FEM code.

The main novel aspects of this method are:

- its mesh free character,
- no need for discretization neither in the domain nor in the boundary,
- same procedure and expression for different geometries of plate,
- solution is obtained performing line integrals only.

2. Preliminary concepts on plate bending problems

In this section some well-known concepts of the classical theory plate bending problems are presented for sake of clarity as well as for introducing appropriate symbols.

Consider a homogeneous isotropic thin plate of constant rigidity D, the governing differential equation [21] in terms of deflection $w(x,y)$ is the biharmonic equation

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)=\frac{q}{D} \tag{1}$$

where q is the out of plane load, $D=Et^3/12(1-\nu^2)$ is the plate flexural rigidity, depending on the modulus of Elasticity E, on the plate thickness t and on the Poisson's ratio ν. Further, Jin et al. [20] decomposed this biharmonic equation into two Poisson's equations as

$$\left(\frac{\partial^2 M}{\partial x^2}+\frac{\partial^2 M}{\partial y^2}\right)=-q \tag{2a}$$

$$\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)=-\frac{M}{D} \tag{2b}$$

being the flexural moment $M=(M_x+M_y)/(1+\nu)$.

If the lateral load q vanishes, as happens with plates loaded only on their boundaries (Fig. 1) then Eqs. (2a) and (2b) become

$$\left(\frac{\partial^2 M}{\partial x^2}+\frac{\partial^2 M}{\partial y^2}\right)=0 \tag{3a}$$

$$\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)=-\frac{M}{D} \tag{3b}$$

Moreover in [22] it has been demonstrated that, for simply supported polygonal plates bent by moments M_n uniformly distributed along the boundary, the following relation holds

$$M=M_n \tag{4}$$

in turn, Eq. (3b) reverts to

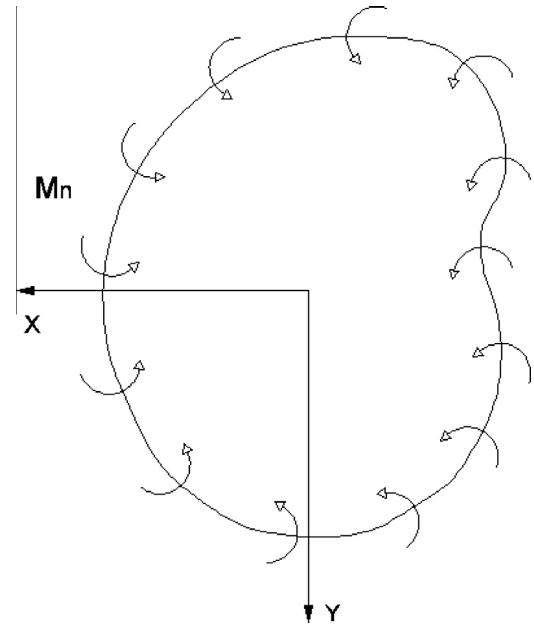

Fig. 1. Simply supported plate loaded by uniformly distributed edge moments M_n.

$$\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)=-\frac{M_n}{D}. \tag{5}$$

3. An innovative formulation for the bending of simply supported plates by Line Element-less Method (LEM)

Looking at Eq. (5) it is apparent the analogy between the bending of simply supported plates loaded by uniformly distributed edge moments and the torsion of a beam [16–18], in fact setting $2G\theta=M_n/D$ (being G the shear modulus and θ the twist rotation of a beam cross-section per unit length) it leads to $\psi(x,y)=w(x,y)$ that is the Prandtl function in a twisted beam is analogous at the deflection function in a simply supported plates loaded by uniformly distributed edge moments. Based on this analogy, LEM for torsion problems has been used for solving plate in bending [19]. Specifically through LEM the shear stress is expressed in series of harmonic polynomials [10,12] and once the shear stress is evaluated then the Prandtl function is determined. In [19] the above procedure has been applied and so the deflection function is evaluated after performing the Prandtl function.

In this paper, it is introduced an innovative expression for the deflection through directly a simple series expansion in terms of harmonic polynomials, in such a way to give a proper method for bending plate without resorting to the analogy with twisted beam.

To aim at this, firstly, the harmonic polynomials P_k and Q_k will be introduced to capture the deflection function $w(x,y)$ of a simply supported plate loaded by uniformly distributed edge moments M_n.

Defining the harmonic polynomials P_k and Q_k as follows:

$$P_k(x,y)=Re(x+iy)^k; \quad Q_k(x,y)=Im(x+iy)^k \tag{6a, b}$$

or recursively as

$$P_k(x,y)=P_{k-1}x-Q_{k-1}y; \quad Q_k(x,y)=Q_{k-1}x+P_{k-1}y \qquad k>0 \tag{6c, d}$$

with $P_0=1,\ Q_0=0,\ P_1=x,\ Q_1=y$. The derivative of the harmonic polynomials are

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