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Advances in co-volume mesh generation and mesh optimisation techniques

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ABSTRACT

This paper introduces developments in modified techniques for the generation of unstructured, non-uniform, dual orthogonal meshes which are suitable for use with co-volume solution schemes. Two new mesh generation techniques, a modified advancing front technique and an octree-Delaunay algorithm, are coupled with a mesh optimisation algorithm. When using a Delaunay–Voronoi dual, to construct mutually orthogonal meshes for co-volume schemes, it is essential to minimise the number of Delaunay elements which do not contain their Voronoi vertex. These new techniques provide an improvement over previous approaches, as they produce meshes in which the number of elements that do not contain their Voronoi vertex is reduced. In particular, it is found that the optimisation algorithm, which could be applied to any mesh cosmetics problem, is very effective, regardless of the quality of the initial mesh. This is illustrated by applying the proposed approach to a number of complex industrial aerospace geometries.

1. Introduction

Computational methods are widely employed in a variety of different application areas. For practical simulations, the requirement of modelling complex geometries means that unstructured mesh methods are particularly attractive, as fully automatic unstructured mesh generation procedures are now widely available. Popular algorithms implemented on unstructured meshes fall into the categories of finite volume and finite elements methods. These algorithms are, generally, low order and often require a significant computational resource to perform accurate simulations involving industrial geometries. In contrast, co-volume techniques exhibit a high degree of computational efficiency, in terms of both CPU and memory requirements, on structured dual orthogonal Cartesian meshes. The marker and cell (MAC) algorithm [3] and the Yee scheme [21] are examples of co-volume methods that have been widely employed for the solution of the Navier Stokes and Maxwell's equations respectively. A basic requirement for the successful implementation of the co-volume scheme is the existence of two, high quality, mutually orthogonal meshes. For an unstructured mesh implementation, the obvious dual mesh choice is the Delaunay–Voronoi diagram. Detailed mesh requirements for co-volume schemes are presented in Section 1.1. Despite the fact that real progress has been achieved in unstructured mesh generation methods over the last two decades, generating suitable co-volume meshes in complex shaped domains is still an open problem. This is due to the difficulties encountered when attempting to generate sufficiently smooth, non-uniform, high quality dual meshes for such problems. Standard mesh generation methods are designed to create high quality Delaunay triangulations, but do not attempt to provide a high quality dual Voronoi mesh. Previous attempts at solving the problem of co-volume mesh generation are discussed in Section 1.2. These techniques can only produce uniform meshes of suitable quality.

For the simulation of geometries that contain regions with high curvature and singularities, non-uniform meshes are often used to capture the complex variation of the solution field. In such cases, the quality of the generated elements depends upon the gradation of the spacing function. This paper presents a number of new techniques designed to generate non-uniform meshes for co-volume solvers. We start from a Delaunay triangulation of the boundary surface and split the generation of the domain into a region adjacent to the boundaries and a free space region. In the free space region, two approaches are introduced to generate meshes and compared to the standard automatic Delaunay sub division method. The first approach is based upon the recursive insertion of ideal lattice points, that locally satisfy the demands of the spacing distribution function, into the Delaunay generated mesh of the

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boundary points. In the second scheme, an octree-Delaunay algorithm is utilised to generate meshes with properties that are close to those of the ideal mesh. In the region adjacent to the boundary, we introduce a modified advancing front technique, in which points are located in such a way that ideal meshes can be recovered in the case of a uniform mesh. Finally, the quality of the resulting mesh is improved by the use of a mesh optimisation scheme. The feasibility of generating suitable meshes in this manner, around increasingly complex geometries, is presented. To demonstrate the validity of the meshes, we present numerical examples of the scattering of electromagnetic waves by a complex 3D object. This example shows the efficiency and accuracy that can be achieved by a co-volume method utilising the proposed meshing scheme.

1.1. Co-volume mesh requirements

Co-volume schemes were initially devised for structured grids and are applied to coupled system of equations such as the Maxwell equations and the Navier–Stokes equations. The unknowns are staggered in space, i.e. some of the unknowns are evaluated at the centre of the cell and the other unknowns are evaluated on the edges of the cell. Finite difference approximation is then used and the solution is advanced in time in a staggered manner, with some of the unknowns evaluated at time n and the others evaluated at time $n+1$. This can be interpreted as using two structured grids which are staggered and orthogonal. On structured grids, a primal mesh is first generated to cover the domain. The dual mesh is then constructed by joining the centre of each cell. This construction guarantees the required orthogonality for finite difference schemes and ensures that the quality of the primal and dual meshes are identical.

The generalisation of the method to unstructured meshes utilises the Delaunay mesh as the primal mesh and its Voronoi mesh as the dual [9]. The Voronoi vertices are the circumcentres of the corresponding Delaunay elements, which guarantees the required orthogonality. However, it is possible for the circumcentre to lie outside its corresponding element. In this case the staggering in space will not be guaranteed. In addition, to replicate the second order accuracy these schemes exhibit on structured grids, the Delaunay and Voronoi grids must intersect each other, i.e. the nodes of the dual mesh should coincide with the centroid of corresponding primal mesh and the dual mesh edges should pass through the centroids of the corresponding primal mesh faces [10]. In two and three dimensional meshes the Voronoi edge by definition intersect the Delaunay edge/face at its centroid, but this construction does not guarantee that the Delaunay face is a bisector of the corresponding Voronoi edge. Furthermore, the time step for an explicit scheme is directly proportional to the smallest edge of the primal and the dual meshes. While this length does not change in structured grids, in unstructured meshes, it is possible for Voronoi edges to vanish, i.e. for two or more elements to share the same circumcentre. Of these two requirements, ensuring that circumcentres lie inside their corresponding elements will be prioritised at the expense of mutual bisection. This is because elements with their circumcentre outside can cause stability problems if located in areas of high gradient in the solution field. Whereas a deviation from mutual bisection results in a local loss of second order accuracy.

Only a mesh made up of equilateral tetrahedra, in which all faces are equilateral triangles, guarantees the quality of the primal and dual meshes that is found in structured grids. However, in contrast to two dimensions, the equilateral tetrahedron is not a space filling element. Hence, a three dimensional analogue of a space filling ideal mesh will consists of equal non-equilateral tetrahedra. Each face in such a mesh will be an isosceles triangle, with one side of length l_D^{long} and two shorter sides of length $l_D^{\text{short}} = \sqrt{3}/2\, l_D^{\text{long}}$. Six such tetrahedra form a parallelepiped tiling the space, as illustrated in Fig. 1. It can be shown that this configuration maximises the length of the minimum Voronoi edge for a fixed element size. All Voronoi edges have the same length $l_V \approx 0.38\,\delta$ where $\delta \equiv \langle l_D \rangle = (3\, l_D^{\text{long}} + 4\, l_D^{\text{short}})/7 \approx 0.92\, l_D^{\text{long}}$. This configuration guarantees that every node has a connectivity index of 14 and each vertex has a maximum angle of 70.5°.

However, the fact some of the element dihedral angles are equal to 90°, makes the element very sensitive to deformation, which become a barrier for the generation of non-uniform meshes. In the case of non-uniform meshes, in order to minimise the deviation of the circumcentre from the barycentre, the requirement that the dual edge has to cross the corresponding element face at the centroid has to be relaxed. This enables the use of the power diagram to locate a new circumcentre, based on a weight associated with each of the element nodes [19]. Furthermore, co-volume schemes are not restricted to using tetrahedral elements, which means that tetrahedral elements can be automatically merged into a polyhedron if the length of the dual edge is below a user specified tolerance. This will reduce the number of elements that have circumcentre located outside their elements and remove any short Voronoi edges that reduce the stability constraint of the scheme.

1.2. Existing mesh generation techniques

To date, most mesh generation software is aimed at generating high quality primal meshes and pays no attention to the quality of the dual mesh. To ensure that the primal mesh and the dual mesh are staggered in space, it is essential to ensure that each Delaunay element contains its Voronoi vertex. A mesh which is staggered in space is termed well centred, as every Delaunay element contains its circumcentre [13]. In two dimensions, this can be achieved by utilising mesh optimisation techniques that are designed to eliminate obtuse triangles [10,11,15]. However, the extension of these techniques into three dimensions is not trivial and does not guarantee the desired outcome.

To address the problem of generating suitable meshes for co-volume techniques, a stitching method was proposed in two dimensions [9]. In this approach, the problem of triangulating a domain of complicated shape is split into a set of relatively simple local triangulations. Each local mesh is constructed with properties which are close to those of an ideal mesh and these local meshes are combined, to form a consistent mesh, using a stitching

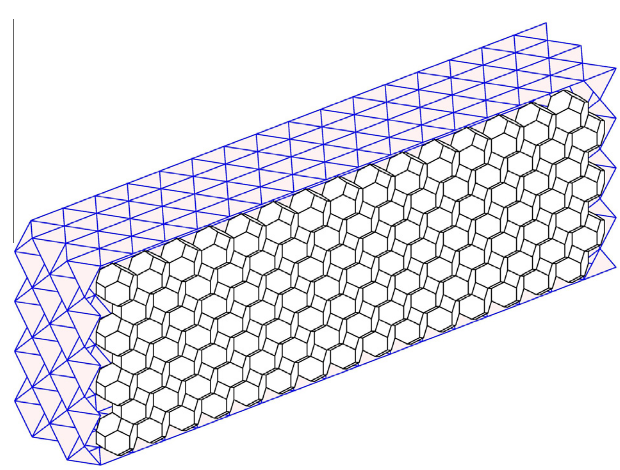

Fig. 1. Detail of a mesh of ideal tetrahedral elements, showing the surface Delaunay faces and the internal Voronoi cells.

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