Computers and Structures 179 (2017) 64–74

Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

Coupling vibration analysis of rotating three-dimensional cantilever beam

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ARTICLE INFO

Article history:
Received 15 February 2016
Accepted 29 October 2016

Keywords:
Rotating beam
Rotating speed limit
Steady-state axial deformation
Chord-wise bending
Flap-wise bending

ABSTRACT

In this study, the coupling equations of motion of a rotating three-dimensional cantilever beam are established to study the effects of Coriolis term and steady-state axial deformation on coupling vibration. In contrast to a previously published method, the present method uses fully nonlinear Green strain–displacement relationships to derive the coupling terms in the equations of motion. The numerical results obtained within the rotating speed limit show that the steady-state axial deformation has considerable effect on the chord-wise bending frequency but not on the flap-wise bending frequency. Moreover, the Coriolis term does not significantly affect the chord-wise bending frequency.

1. Introduction

The dynamics of rotating cantilever beams have drawn much attention because such beams are widely used in engineering applications such as flexible manipulators, thin and long wind turbine blades, high-speed rotating helicopter rotor blades, and turbine engine blades. Because of rotational motion, the natural vibration characteristics of the rotating beams are considerably different from those of beams without rotational motion. For accurate operation and precise control of such rotating beams, an accurate and reliable dynamic model needs to be established.

Accurate modelling of the geometric stiffening effect is one of the key elements in the dynamic modelling of a rotating cantilever beam. Several modelling methods have been proposed, such as the following: (1) a method that directly provides axial centrifugal loads [1–4], (2) a method that considers the foreshortening effect in the longitudinal displacement [5–18], and (3) absolute nodal coordinate formulation [19,20]. Likins et al. [1] and Simo and Vu-Quoc [2] initially developed the first method. Kaya and Ozgumus [3] used this method to study the bending–torsion coupling problem of a rotating Timoshenko cantilever beam and analysed the influence of the bending–torsion coupling effect and Coriolis term on structural natural frequencies and mode shapes. Banerjee and Kennedy [4] developed the in-plane free vibration differential equation of motion and analysed the free vibration characteristics by using the series solution of the Frobenius method and the Wittrick–Williams algorithm. Their analysis was focused on the effects of the Coriolis term and hub radius.

The method based on the foreshortening effect considers the effect of longitudinal shrinkage caused by transverse bending in the calculation of the longitudinal displacement of a rotating cantilever beam to obtain the geometric stiffening terms. Vigneron [5] described this longitudinal displacement and provided a detailed derivation of the method to obtain the geometric stiffening terms using the foreshortening effect [6]. By comparing the equation based on this derivation to the transverse bending vibrational equation obtained by Likins et al. [1], Vigneron pointed out that the method based on the foreshortening effect has better versatility. Kane et al. [7] presented a dynamic modelling method for rotating beams that was different from Vigneron [6]. Kane's modelling method described the longitudinal displacement using the coordinates after deformation, considering factors such as stretching, bending, and shearing. It also avoided the impact of early cut off axial strain on later results. In Kane's modelling method [7], the coordinates after deformation are used to describe longitudinal displacement whereas the coordinates before deformation are used to separate the variables. Addressing this inconsistency, Hanagud and Sarkar [8] provided a more general description method that uses the coordinates before deformation to describe longitudinal displacement and performs variable separation for the same coordinates. Sharf [9] classified and discussed the

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http://dx.doi.org/10.1016/j.compstruc.2016.10.024
0045-7949/

Nomenclature

A	cross-section area, m^2
$\mathbf{B}^j$	the Boolean assembly matrix
E	Young's modulus, N m^2
$\mathbf{G}^j$	gyroscopic matrix
$\mathbf{H}$	coefficient matrix
I_y, I_z	second moment of area of the cross-section, m^4
i	an imaginary number
$\mathbf{K}^j_{\mathrm{Axial}}$	stretching stiffness matrix
$\mathbf{K}^j_{\mathrm{Chord}}$, $\mathbf{K}^j_{\mathrm{Flap}}$	transverse bending stiffness matrix
l	the length of the element, m
L	length of beam in the undeformed configuration, m
m	the order of axial stretching circular frequency
$\mathbf{M}^j_{\mathrm{Axial}}$	mass matrix
$\mathbf{M}^j_{\mathrm{Chord}}$, $\mathbf{M}^j_{\mathrm{Flap}}$	mass matrix
n	the total number of finite elements
$\mathbf{N}_{\mathrm{Axial}}$	shape function vector
$\mathbf{N}_{\mathrm{Chord}}$, $\mathbf{N}_{\mathrm{Flap}}$	shape function vector
$\mathbf{p}_{\mathrm{Flap}}$	mode shape vector
$\hat{\mathbf{p}}_{\mathrm{Chord}}$	mass-normalized mode shape vector
P_0	arbitrary point on the centerline of the beam in undeformed configuration
P	point P_0 in the deformed configuration
$\mathbf{q}^j_{\mathrm{Axial}}$	nodal coordinate vector
$\mathbf{q}^j_{\mathrm{Chord}}$, $\mathbf{q}^j_{\mathrm{Flap}}$	nodal coordinate vector
$\mathbf{r}$	global position vector of the point P, m
$\mathbf{R}$	planar rotation matrix
t	time, s
T	kinetic energy
u_1, u_2, u_3	total longitudinal and transverse displacements, m
$\mathbf{u}_0$	local position vector of an arbitrary point P_0, m
$\mathbf{u}_f$	displacement vector, m
U	strain energy
$\mathbf{v}$	global velocity vector of the point P, m s^{-1}
V	volume of beam, m^3
w_1, w_2, w_3	axial, chord-wise and flap-wise bending displacements, m
w_{1ry}, w_{1rz}	longitudinal displacement caused by rotating of the cross-section, m
w_{cy}, w_{cz}	longitudinal shrinkage caused by the transverse bending displacement, m
w_{1s}	time-independent steady-state axial deformation, m
w_{1v}	time-dependent axial vibration response, m
x	horizontal component of P_0, m
$\bar{x}$	the coordinate of the point on the centre line of the beam element, m
ε_{11}	longitudinal normal strain
$\boldsymbol{\eta}$	state vector
θ	angle of rotation, rad
$\dot{\theta}_l$	rotating speed limit, rad s^{-1}
ρ	mass per unit volume, kg m^{-3}
σ_l	elastic limit, Pa
ω_v	axial stretching circular frequency, rad s^{-1}
ω_{v0}	axial stretching circular frequency without rotational motion, rad s^{-1}
ω_{Flap}	flap-wise bending circular frequency, rad s^{-1}
ω_{Chord}	chord-wise bending circular frequency, rad s^{-1}
j	superscript, the number of finite elements
"$\cdot$"	the derivative with respect to time t
"′"	the derivative with respect to coordinate x

researches of Kane et al. [7], Likins et al. [1], Vigneron [6], and Simo and Vu-Quoc [2] in detail in terms of the geometric stiffening effect and axial displacement problems and compared the differences among the different modelling methods. El-Absy and Shabana [10] investigated the effect of the geometric stiffening term on the stability of the elastic and rigid body modes, and then, using several models, examined whether longitudinal shrinkage is included in both the elastic and inertia forces. Yoo and Shin [11] studied the vibration problem in an in-plane rotating three-dimensional cantilever beam following the same perspective as in the case of Kane et al. [7]. They mainly focused on the impact of Coriolis term on the structural bending vibration characteristics and determined corresponding Southwell parameters for different ratios between the hub's radius and beam length. Li et al. [12] also considered the effect of Coriolis term on the bending frequency. Liu et al. [13] established the finite element formulation of rotating flexible beam, and analysed the effects of inertia forces and elastic forces related to the coupling displacement on system dynamic behaviour, respectively. Chung and Yoo [14] established the finite element formulation of rotating cantilever beam and compared the bending frequency obtained by using the assumed modes method from Ref. [11]. Tsai et al. [15] studied the dynamic problem of rotating inclined Euler beam by co-rotational finite element formulation combined with the rotating frame method, and analysed the effect of inclination angle on the bending frequency with different radius of the hub, slenderness ratios and rotating speed. Kwon et al. [16] established a structural bending vibrational equation for wind turbine blades without considering the effect of axial displacement using Kane's method. The study discussed the effect of structural gravitational force and the tilt and pitch angle of the blade relative to the generator shaft on structural displacement and dynamic stability characteristics. Duan et al. [17] discussed the global impact dynamic problems of rotating cantilever beam. The dynamic model was established by considering the longitudinal foreshortening effect caused by the transverse bending displacement, and then analysed the impact process using the continuous contact force method and contact constraint method, respectively. The comparison between the experimental verification and the former two methods showed the advantages and weaknesses of the two methods. Fang and Zhou [18] studied the free vibration problem of the rotating axially functionally graded-tapered beam. The transverse displacement amplitude function was expressed in the form of Chebyshev polynomial series. The effects of material gradient index, rotating speed ratio, hub radius ratio and taper ratio on bending frequency were analysed.

Berzeri and Shabana [19] introduced a modelling theory based on the absolute nodal coordinate formulation in detail. They characterised the complex forms of longitudinal and transverse forces by deriving different models based on different assumptions and provided explanations for each of them. Berzeri and Shabana [20] studied the modelling problem of a rotating cantilever beam using the characteristics of a floating coordinate frame by taking advantage of the absolute nodal coordinate formulation. The study gave a generalized characteristic equation based on a completely different perspective, analysed the frequency and mode shape problem of the structure, and compared the frequency from the Southwell equation.

Other researchers also investigated the relevant structural dynamic problems. Warminski and Balthazar [21] studied the nonlinear vibrations of a rotating light beam by using the Galerkin's