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Higher order analysis of thin-walled beams with axially varying quadrilateral cross sections

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ABSTRACT

A thin-walled beam finite element with a varying quadrilateral cross section is formulated based on a higher order beam theory. For the calculation of distortions, the beam frame approach, which models the cross section by using two-dimensional Euler beams, is used. Distortions induced by the Poisson's effect and warpings are analytically derived. Three-dimensional displacements at an arbitrary point of a present beam element can be described by interpolating three-dimensional displacements at the end sections. Straight and curved thin-walled beams with varying cross sections are solved to show the validity of the proposed approach.

1. Introduction

For a one-dimensional analysis of a thin-walled beam, it is essential to consider higher order cross-section deformations such as warping and distortions. Three-dimensional displacements of a thin-walled beam are represented by linear combination of one-dimensional deformation measures such as warping and distortions as well as conventional rigid body cross-section deformations. Three-dimensional elasticity equations can be reduced to one-dimensional governing equations in terms of the cross-section deformation measures by integrating the products of the cross-section deformation modes or their derivative forms. The most classical approach for a higher order beam analysis can be found in Vlasov [1], where the warping mode of a thin-walled beam with an open cross section was obtained by assuming negligible shear strain along the middle line of the cross section.

In von Kármán and Christensen [2], thin-walled beam problems with closed cross sections were solved by separately calculating the shear flows by uniform and nonuniform twist. In the case of nonuniform twist, the bending of a longitudinal strip was considered in order to calculate the axial strain. Thereby, the normal stress and shear stress could be obtained by using the constitutive and equilibrium equations, respectively. More recently, the generalized beam theory employed enriched cross-section deformations with both out-of-plane warping and in-plane distortions to solve buckling problems of open and closed thin-walled beams [3,4,5,6]. In Ferradi and Cespedes [7], cross-section distortions were obtained by solving the eigenvalue problem of a cross section, and for each distortion, the corresponding warping was calculated by using an iterative equilibrium approach. Carrera et al. [8] proposed the unified formulation in which displacement fields are refined by adopting Taylor-type expansions with respect to cross-sectional coordinates. Based on their unified formulation, higher order models of distortions and warpings could be accounted for straightforwardly without predefining specific cross-section shape functions in static analysis [9] and linear buckling analysis [10]. Without employing kinematic assumptions, Yu et al. [11] mathematically derived one-dimensional beam equations by applying the variational asymptotic method to the energy functional. They solved geometrically nonlinear composite beam problems by using the variational asymptotic method.

Although higher order beam analysis methods have been actively researched to accurately predict static and dynamic responses of thin-walled beams, their applications are somewhat limited in designs for complex frame structures such as automotive bodies. The difficulties develop due to geometric complexities such as: (1) generally-shaped cross sections varying rapidly along the axial directions of beams; and (2) thin-walled beams connected at joints while cross-section deformations show very

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complex coupling behaviors. To deal with the difficulty of the joint connection, the most popular approach is to attach a rotational spring at the joint so that the flexibility additionally introduced by coupled deformations can be paid for [12–14]. However, it is not possible to find effective spring stiffness which is consistently applicable to general joint connections. Therefore, the spring stiffness must be recalculated for every new joint connection by using three-dimensional shell analysis [15]. The continuity of the three-dimensional displacements, which are based on a higher order beam theory, is considered at a joint in Jang et al. [16], Jang et al. [17] and Choi et al. [18]. Those research efforts were undertaken to find the matching conditions for a joint among the one-dimensional field variables. Although their method was restricted to rectangular cross-sectioned thin-walled beams, the joint stiffness could be effectively evaluated based on the higher order beam analysis without using artificial springs.

In this investigation, the analysis of thin-walled beams with general quadrilateral cross sections by Kim and Kim [19] is extended for varying cross-section problems to resolve the first difficulty of the geometric complexities mentioned above. The quadrilateral cross section of a thin-walled beam with uniform wall thickness varies arbitrarily along the axial direction of the beam, which is not necessarily straight. Although torsional distortion of a quadrilateral thin-walled cross-section was analytically derived in Kim and Kim [19], other higher order distortions required for solving varying cross-section problems cannot be obtained by the analytic approach. Therefore, the beam frame model in Jang et al. [20] is employed where distortion modes are obtained by solving the eigenvalue problem of the two-dimensional beam frame model. In joint problems of rectangular thin-walled beams by Choi and Kim [21], it was shown that distortions induced by Poisson's effect, hereafter called Poisson distortions, are coupled with warping at the joint. The Poisson distortions are also coupled with bending/shear rotations so the solution is very stiffly estimated without requiring the inclusion of the Poisson distortions as field variables. However, because wall-stretching deformations are dominant in Poisson distortions, they cannot be obtained by using the approach of the beam frame model.

In this study, Poisson distortions are analytically calculated by considering the equilibrium of the plane stress states. In addition, warping modes are derived as an integration form from the derivatives of corresponding distortions. Orthogonalities with respect to rigid body cross-section deformations are imposed as constraints when the higher order modes are analytically or numerically calculated. Three-dimensional displacements at an arbitrary point of a beam can be interpolated by using those at discrete cross sections (or nodes) where the analyses for higher order modes are conducted. Calculating strain components with respect to local coordinates to utilize plane stress conditions can be performed by following the similar procedure for the formulation of the shell elements.

The proposed higher order beam elements are applied to the solution of the varying cross-section problems with trapezoidal and general quadrilateral cross sections. Curved beam problems as well as straight beam problems are presented to show the effectiveness.

2. Cross-section shape functions for general quadrilateral cross-sections

In higher-order beam theories, three-dimensional displacements at an arbitrary point of a beam are approximated by using cross-sectional deformations including higher-order modes and one-dimensional deformation measures. The strain and stress fields are derived according to strain-displacement relations and constitutive equations. Differential equations of equilibrium can be found in terms of the one-dimensional deformation measures by following the principle of the minimum potential energy for the calculated strains and stresses.

In Fig. 1, a general quadrilateral thin-walled cross-section is illustrated. On the center line of the cross section, the three-dimensional displacements are described as

$$u_p(s,z) = \sum_{i=1}^{N_\psi} \psi_p^i(s) d_i(z) = \boldsymbol{\psi}_p \mathbf{d}, (p = n, s, z) \tag{1}$$

where s and n are the tangential and outward normal coordinates on the cross-section contour, and z is the axial coordinate of a beam. In Eq. (1), $\psi_p^i(s)$ are shape functions for the cross-section deformation, $d_i(z)$ are the corresponding one-dimensional deformation measures (or field variables), and N_ψ is the number of the shape functions. Note that higher order cross-section deformations as well as conventional rigid body cross-section deformations are included in ψ_p^i. For higher-order deformations, warping and distortion are considered separately for out-of-plane and in-plane deformations, respectively. The shape functions for rigid body cross-section deformations are:

$$\psi_n^{U_x}(s_j) = \sin(\alpha_j), \psi_s^{U_x}(s_j) = \cos(\alpha_j), \tag{2a}$$
$$\psi_n^{U_y}(s_j) = -\cos(\alpha_j), \psi_s^{U_y}(s_j) = \sin(\alpha_j), \tag{2b}$$
$$\psi_z^{U_z}(s_j) = 1, \tag{2c}$$
$$\psi_z^{\theta_x}(s_j) = \frac{y_{j+1} - y_j}{l_j} s_j + y_j - y_c, (y_5 = y_1) \tag{2d}$$
$$\psi_z^{\theta_y}(s_j) = -\frac{x_{j+1} - x_j}{l_j} s_j - x_j + x_c, (x_5 = x_1) \tag{2e}$$
$$\psi_n^{\theta_z}(s_j) = s_j - l_j, \psi_s^{\theta_z}(s_j) = r_j, \tag{2f}$$

where U_q and θ_q $(q = x, y, z)$ denote rigid body translations and rotations of the cross section in the q-direction, respectively. In the above, it is convenient to express shape functions in terms of edge-wisely defined tangential coordinate s_j where the subscript j is the edge index $(j = 1, 2, 3, 4)$. In Eqs. (2a) and (2b), α_j is the angle between edge j and the x-axis, (x_j, y_j), coordinates of corner j, and (x_c, y_c), coordinates of the geometric center (see Fig. 1). In Eq. (2f), r_j is the distance from the shear center to edge j, and l_j is the distance from corner j to the point N_j, which is defined as the intersection between edge j and the normal line from the shear center to

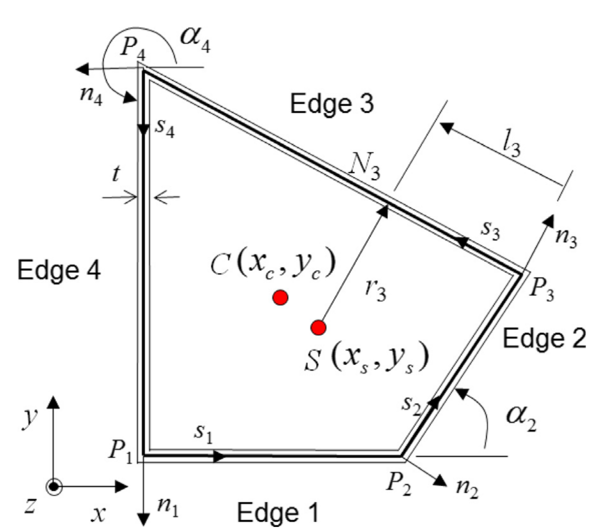

Fig. 1. General quadrilateral cross section of a thin-walled beam.