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# Stability analysis and evaluation of staggered coupled analysis methods for electromagnetic and structural coupled finite element analysis

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ABSTRACT

The numerical stability of electromagnetic and structural coupled analysis methods is examined in terms of the spectral radius for various methods combined with a coupled algorithm, time integration methods for structural analysis and eddy current analysis, and coupled effect evaluation methods. The coupled analysis method with the conventional serial staggered algorithm, generalized–$\alpha$ method for structure, backward difference method for eddy current, and coupled effect evaluation using previous time step results is seen to be the most suitable from the viewpoints of stability and computing time. The stability of the conventional parallel staggered algorithm is found to be much improved if the generalized–$\alpha$ method is used for structural analysis.

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## 1. Introduction

The use of coupled finite element analyses, such as fluid–structure interaction analysis and electromagnetic–structural coupled analysis, is increasing in the design of mechanical components. Coupled finite element analysis methods are classified as monolithic or partitioned methods. In monolithic methods, the coupled finite element equations are obtained by combining the finite element equations for multiphysics phenomena and then solving them. However, a high computational cost incurs since the matrix size becomes rather large. In partitioned methods, multiple finite element equations are separately solved. Because the computational cost of the partitioned method is low and using single-phenomenon analysis codes is easy, such methods are used in several coupled analyses. However, numerical instability may occur owing to the time lag in coupled effect evaluation even if the time integration method for each phenomenon is unconditionally stable.

Many studies on partitioned methods, including both staggered methods and those with iterations, have been conducted for fluid–structure interaction problems. In addition to the conventional serial staggered (CSS) algorithm, which is widely used for staggered analysis, several coupled algorithms have been proposed such as the conventional parallel staggered (CPS) algorithm, improved serial staggered (ISS) algorithm, and improved parallel staggered algorithm and their numerical stability, resulting accuracy, and computing time have been previously discussed [1–3]. In partitioned methods with iterations, such as the block Jacobi method and block Gauss–Seidel method, the stability is improved through iterative procedures. Among other partitioned methods using iterative procedures, the block Newton method has been proposed; its solution convergence and computing time have been compared with those of the block Gauss–Seidel method [4,5]. A partitioned method using reduced order models for fluid and structural problems has been proposed, and its solution convergence has been compared with that of the block Newton method [6].

Magnetic damping vibration is one type of an electromagnetic and structural coupled problem. Studies have focused on magnetic damping vibration analysis, which is required for the design of conductive structures located in a strong magnetic field, such as those in future fusion reactors or magnetically levitated vehicles. Several coupled analysis methods have been compared for magnetic damping vibration with the bending mode [7] and with the bending and torsional modes [8] from the viewpoint of modeling, formulation, type of element, and time integration method. In the past few years, the geometrical nonlinearity of magnetic damping vibration has been discussed [9]; a coupled analysis method using a Lagrangian approach has been proposed [10]. However, the CSS algorithm was almost always used in magnetic damping vibration analysis, and few studies have focused on coupled algorithms. Moreover, numerical instability occurs in magnetic

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damping vibration analysis even if unconditionally stable time integration methods are used, especially for strong coupling conditions, such as a high-intensity magnetic field.

Many studies of numerical stability and stabilization techniques have been conducted for single-phenomenon finite element analysis. However, coupled analysis often becomes unstable, even if these stabilized techniques are used. The numerical stability of coupled analyses has also been studied. In a study of fluid–structure interaction analysis, the numerical stability and solution accuracy were examined in relation to conservation [11]. The numerical stability was compared among iterative coupling schemes using Fourier error analysis [12,13]. The effects of interpolating the traction force on numerical stability have been investigated in terms of stability analysis using the spectral radius of a one-dimensional damped spring–mass model for the fluid–structure interaction problem. The numerical stability was also compared using coupled finite element analysis of the flow-induced oscillation of a flexible beam [14]. A coupled analysis method stabilized by the generalized–$\alpha$ method was proposed for a fluid–structure interaction problem, and stability analysis was performed using the spectral radius of a one-dimensional damped spring–mass model. The validity of the stability analysis results was confirmed using coupled finite element analysis of the oscillation problem [15].

Many studies on fluid–structure interaction analysis have considered not only the coupled analysis method but also the numerical stability; however, most studies have focused on only a qualitative comparison without performing stability analysis. Even if stability analyses were performed, they would have been limited to particular cases, such as one-dimensional coupled problems only, discussing the stability of the proposed coupled analysis method and a few combinations of the time integration method and the coupled effect evaluation method. As for the magnetic damping vibration analysis, numerical instability has attracted little attention.

In this study, a stability evaluation method is proposed for the coupled finite element analysis of magnetic damping vibration. In this method, the stability is evaluated using the spectral radius obtained from the coupled eigenmode and the time integration scheme. Next, the numerical stability is systematically and quantitatively examined in terms of the stable region for various coupled analysis methods that are combined with a coupled algorithm, time integration methods for structural analysis and eddy current analysis, and coupled effect evaluation methods. Finally, the computing time required to obtain solutions that are both stable and convergent is compared in order to evaluate the practicality of the different coupled analysis methods.

## 2. Coupled analysis method for electromagnetic and structural coupled problems

### 2.1. Magnetic damping vibration

Magnetic damping vibration occurs in a conductive structure located in a magnetic field. The Lorentz force and electromotive force have a coupled effect on the vibration and eddy current, respectively.

Fig. 1 shows the magnetic damping vibration of a conductive plate clamped at one end and placed in a horizontal magnetic field $\boldsymbol{B}_x$ and vertical magnetic field $\boldsymbol{B}_z$. The plate is vibrated by the Lorentz force $\boldsymbol{J} \times \boldsymbol{B}_x$, as shown in Fig. 1(a), where $\boldsymbol{J}$ is the eddy current produced by the time change of $\boldsymbol{B}_z$. While the plate is vibrating with deformation velocity $\dot{\boldsymbol{u}}$, the electromotive force $\dot{\boldsymbol{u}} \times \boldsymbol{B}_x$ reduces the eddy current and vibration, as shown in Fig. 1(b).

### 2.2. Finite element equation

The $T$ method is used for eddy current analysis of the magnetic damping vibration problem of a thin shell structure [16]. The matrix equation of the eddy current analysis is expressed using the nodal point normal component $\boldsymbol{T}$ of the current vector potential and nodal point deformation vector $\mathbf{u}$:

$$\mathbf{U}\dot{\boldsymbol{T}} + \mathbf{R}\boldsymbol{T} = \mathbf{C_e}\dot{\mathbf{u}} + \dot{\boldsymbol{B}}^{\boldsymbol{ex}}. \tag{1}$$

Here, $\mathbf{U}, \mathbf{R}, \mathbf{C_e}$, and $\dot{\boldsymbol{B}}^{\boldsymbol{ex}}$ are the inductance matrix, the resistance matrix, the coupling sub-matrix of the electromotive force, and the time-varying external magnetic field, respectively.

The matrix equation of the structural analysis is expressed by

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{C_s}\boldsymbol{T} + \boldsymbol{F}^{\boldsymbol{ex}}, \tag{2}$$

where $\mathbf{M}, \mathbf{K}, \mathbf{C_s}$, and $\boldsymbol{F}^{\boldsymbol{ex}}$ are the mass matrix, the stiffness matrix, the coupling sub-matrix of the Lorentz force, and the external force, respectively. The 8–node quadratic isoparametric shell element is used in finite element discretization for both eddy current analysis and structural analysis.

### 2.3. Coupled algorithm

Coupled analysis methods for magnetic damping vibration are classified as simultaneous or staggered methods. In a simultaneous method, the coupled finite element equation obtained by combining Eqs. (1) and (2) has been solved [16] and shown to be unconditionally stable [17]. In a staggered method, Eqs. (1) and (2) are solved separately and alternately, and the time integration can be performed using either the direct time integration method or the mode superposition method. However, it is conditionally stable even if unconditionally stable time integration methods are used for each equation since the solution diverges by numerical instability under specific conditions, e.g., according to the intensity of the magnetic field and the time increment. In addition to the CSS algorithm, the CPS algorithm and ISS algorithm have been proposed for use in fluid–structure interaction analysis [1–3]. According to these previous studies, the ISS algorithm is suitable for coupled analysis from the viewpoint of numerical stability and solution accuracy, and the CPS algorithm has weak stability. In this study, the numerical stability, solution convergence, and computing time of these coupled algorithms are discussed for staggered methods of magnetic damping vibration analysis.

Fig. 2 shows the data flow between the eddy current analysis and the structural analysis using the CSS, CPS, and ISS algorithms for magnetic damping vibration analysis. In the CSS algorithm, Eq. (1) for the eddy current analysis is solved using the results from the previous time step of the structural analysis to evaluate the coupling term in Eq. (1). Then, Eq. (2) for the structural analysis is solved using the results of the eddy current analysis to evaluate the coupling term in Eq. (2). In the CPS algorithm, Eq. (1) for the eddy current analysis and Eq. (2) for the structural analysis are solved simultaneously and separately at each time step. The terms for the coupled effect in Eqs. (1) and (2) are evaluated using the results from the previous time step. In the ISS algorithm, Eq. (1) for the eddy current analysis is solved in time $t - \Delta t/2$ first, using the result for $t - \Delta t$ from the structural analysis to evaluate the coupling term. Then, Eq. (2) for the structural analysis is solved in time $t$ using the result for $t - \Delta t/2$ from the eddy current analysis to evaluate the coupling term. Therefore, both coupling terms have a time lag of $\Delta t/2$.

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