Computers and Structures 176 (2016) 13–33

Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

Implementation and validation of a total displacement non-linear homogenization approach for in-plane loaded masonry

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A R T I C L E I N F O

Article history:
Received 31 October 2015
Accepted 5 August 2016

Keywords:
Masonry
In-plane loads
Semi-analytical approach
Compatible model of homogenization
Holonomic non-linear model

A B S T R A C T

Two simple homogenization models suitable for the non-linear analysis of masonry walls in-plane loaded are presented. A rectangular running bond elementary cell is discretized by means of twenty-four constant stress three-noded plane-stress triangular elements and linear two-noded interfaces. Non-linearity is concentrated on mortar reduced to interface, exhibiting a holonomic behavior with softening. The paper shows how the mechanical problem in the unit cell can be characterized by very few displacement/stress variables and how homogenized stress-strain behavior can be evaluated by means of a small-scale system of non-linear equations. At a structural level, it is therefore not necessary to solve a homogenization problem at each load step in each Gauss point and a direct implementation into commercial software as an external user supplied subroutine is straightforward. Non-linear structural analyses are conducted on a variety of different problems, for which experimental and numerical data are available in the literature, in order to show that accurate results can be obtained with a limited computational effort.

1. Introduction

Masonry is a composite material constituted by bricks (or blocks) joined by mortar. The variability of the masonry bond (or arrangement of the bricks), the shape and dimension of the bricks, as well as the quasi-fragile behavior of the constituent materials, make the simulation of masonry still a challenging task. At present, two main approaches are utilized to numerically describe masonry behavior after the elastic limit, which usually is exceeded at low levels of external loads, known in the technical literature as macro-modeling and micro-modeling.

Macro-modeling does not make any distinction between masonry units and joints, averaging the effect of mortar through the formulation of a fictitious continuous material. The literature in this regard is extensive [1–3], with the noticeable example of no-tension material modeling (e.g. [1]), traditionally conceived to deal with non-linear problems exhibiting predominant mode I fracture of the joints (e.g. arches or pillars under rocking) and masonry with good compressive strength, where crushing and orthotropic behavior are not paramount. Macro-modeling allows the rough discretizations necessary for the analysis of large scale structures. Nevertheless, it is difficult to take into account some distinctive aspects of masonry in this approach, such as anisotropy in the inelastic range and the post-peak softening behavior in both tension and compression, unless sophisticated approaches with multiple inelastic parameters are adopted. In this regard, some equivalent macro-models have been presented [2,3], featuring orthotropic elastic–plastic behavior with softening. Theoretically, such approaches are capable of adequately estimating the non-linear masonry behavior along any load combination, even if some limitations may occur in specific cases (see [4] for a detailed discussion). Costly experimental campaigns are needed to consistently evaluate data fitting mechanical coefficients that fully define the models.

The alternative micro-modeling approach is simply characterized by distinct modeling of mortar joints and bricks at structural level. The reduction of joints to interfaces [5–7] helps in limiting variables, especially in the non-linear range, but the approach is computationally demanding and the need of modeling separately bricks and mortar limits its applicability to structural elements and small case studies. Therefore, it can be stated that, at present, the analysis of masonry walls in the inelastic range requires macro-scale computations with finite elements (FEs) [8,9]. In such scenario, homogenization [10–19] is a fair compromise between micro- and macro-modeling, because it allows

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http://dx.doi.org/10.1016/j.compstruc.2016.08.001
0045-7949/

non-linear analyses of large scale structures, still considering the real disposition of bricks and the actual mechanical properties of the constituent materials at a cell level. Clearly, the numerical models to use at structural level should be sufficiently simple, reliable and efficient to allow a quick evaluation of (a) collapse loads, (b) displacements near collapse and (c) post peak behavior of the structures.

Homogenization (or related simplified approaches) consists in extracting a representative element of volume (RVE) that generates the whole structure by repetition, in solving a boundary value problem on the RVE and in substituting the assemblage of bricks and mortar at a structural level with a fictitious orthotropic equivalent material. The most straightforward procedure is the utilization of FEs [13,20], assuming either elasto-plastic or damaging constitutive laws for units and mortar. Nevertheless, the so-called FE^2, i.e. a twofold discretization, the first for the unit cell and the second at structural level, proved to be still too demanding, since the field problem has to be solved numerically for each load step, in each Gauss point. Alternatively, in this paper, a simplified homogenization two-step model is used to analyze masonry walls in-plane loaded. In the first step, masonry is substituted with a macroscopic equivalent material through a so-called compatible identification, belonging to the wide family of the homogenization procedures. The unit cell is meshed by means of 24 triangular constant stress (CST) plane stress elements (bricks) and linear interfaces for mortar joints. Triangular elements are assumed linear elastic, whereas the mechanical response of the interface elements includes two dominant failure modes, namely cracking (mode I) and shear (mode II) or a combination of two (mixed mode). Such elements are equipped with a constitutive relationship referred to as "holonomic", since expressed in terms of normal and tangential tractions σ and τ as a path independent function of the normal and tangential relative displacements at the interface. Both a piecewise linear and an exponential law are implemented, formally identical to an improved version of the Xu-Needleman law and proposed in another context [21–23]. Such cohesive relationships are characterized by a post-peak softening branch, possibly with coupling between normal and shear relationships in the case of the improved Xu-Needleman model.

Two slightly different approaches are compared. The first (Model I) translates the mechanical problem into mathematics by means of a system of a few non-linear equations, which is solved with standard general purpose algorithms. The second (Model II) is a semi-analytical two variables procedure. While semi-analytical homogenization is a method already known and used in periodic fiber-reinforced composites, see e.g. [24], this is one of the first applications for periodic masonry, that at the same time allows a rigorous conservation of anti-periodicity of the stress field and periodicity of displacements.

In the second step, entire masonry walls are analyzed in the inelastic range by means of a commercial FE code where the discretization is constituted by quadrilateral rigid elements and homogenized holonomic tensile-shear springs. It is worth mentioning that most commercial codes can be suitably used at this aim. The procedure is efficient and reliable because: (1) the disadvantages of FE^2 are superseded since the solution in terms of displacements and stresses is found at a cell level with very limited computational effort, using an implementation of the routine adopted at a meso-level to evaluate homogenized quantities directly at structural level; (2) it is not necessary to discretize with refined meshes the elementary cell and hence Gauss point computations are much faster, where only few kinematic stress variables are needed; and (3) the holonomic laws assumed for mortar joint allow for a total displacement formulation of the model, where the only variables entering into the homogenization problem are represented by displacements.

2. The simplified (compatible homogenization) holonomic model

One of the basic concepts of homogenization relies in introducing averaged quantities representing the macroscopic strain and stress tensors (respectively $\mathbf{E}$ and $\mathbf{\Sigma}$) [13,25] on a representative element of volume Y (RVE or elementary cell, Fig. 1), i.e. $\mathbf{E} = \langle \boldsymbol{\varepsilon} \rangle = \frac{1}{A}\int_Y \boldsymbol{\varepsilon}(\mathbf{u})dY$ and $\mathbf{\Sigma} = \langle \boldsymbol{\sigma} \rangle = \frac{1}{A}\int_Y \boldsymbol{\sigma} dY$, where A stands for the area of the elementary cell, $\boldsymbol{\varepsilon}$ and $\boldsymbol{\sigma}$ stand for the local quantities (strains and stresses respectively) and $\langle * \rangle$ is the averaging operator. Periodicity conditions are imposed on the stress field $\boldsymbol{\sigma}$ and the displacement field $\mathbf{u}$, given by:

$$\begin{cases} \mathbf{u} = \mathbf{E}\mathbf{y} + \mathbf{u}^{per} & \mathbf{u}^{per} \text{ on } \partial Y \\ \boldsymbol{\sigma}\mathbf{n} & \text{anti-periodic on } \partial Y \end{cases} \tag{1}$$

where $\mathbf{u}$ is the total displacement field, $\mathbf{u}^{per}$ stands for a periodic displacement field, $\tilde{x} = \{x\ y\ z\}$ is the local frame of reference (see Fig. 1), $\mathbf{E}$ is the homogenized strain tensor and $\mathbf{n}$ is the outward versor of the ∂Y surface.

In the model proposed, which is a simplified homogenization hereby designated as "compatible identification" (as coined in [26], where additional details can be found), joints are reduced to interfaces with zero thickness and bricks are discretized by means of a coarse mesh constituted by three noded plane-stress elements, as schematically sketched in Fig. 1. The choice of meshing 1/4 of the brick through at least 3 triangular elements is due to the need of reproducing the presence of shear stress in the bed joint (element 2 in Fig. 1) in horizontal stretching.

When dealing with the non-linear approach presented hereafter [11], all the non-linearity in the RVE is concentrated on joints reduced to interfaces. With the coarse discretization adopted, 1/4 of the RVE is meshed through 6 CST elements, labeled in Fig. 1 as 1, 2, 3, 1′, 2′, 3′.

Indicating with $\circ^{(n)}$ a stress component belonging to the nth element, the plane stress Cauchy stress tensor inside the nth CST element $\boldsymbol{\sigma}^{(n)}$ is constituted by the components $\sigma_{xx}^{(n)}$ (horizontal stress), $\sigma_{yy}^{(n)}$ (vertical stress) and $\tau^{(n)}$ (shear stress). When dealing with static quantities, equilibrium inside each element is a priori satisfied, $div\boldsymbol{\sigma} = 0$, whereas two equality constraints involving Cauchy stress tensor components of triangles have to be imposed for each internal interface between adjoining elements. In particular, for 1–2 interface, it has to be ensured that the stress vector (normal and tangential component) is equal passing from element 1 to element 2, i.e. $\sigma_{xx}^{(2)} = \sigma_{xx}^{(1)} + \rho(\tau^{(1)} - \tau^{(2)})$ and $\sigma_{yy}^{(2)} = \sigma_{yy}^{(1)} + \rho^{-1}(\tau^{(1)} - \tau^{(2)})$, where ρ is the ratio between the semi-length of the bricks and its height, i.e. $\rho = L/2H$. Analogous equations can be written for 3–2, 3′–2′, 2–2′ and 2′–1′ interfaces.

Assuming that the triangular elements are linear elastic, the following relationship between strains and stresses can be written:

$$\begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \frac{\sigma_{xx}}{E_b} - \frac{\nu_b \sigma_{yy}}{E_b} \\ -\frac{\nu_b \sigma_{xx}}{E_b} + \frac{\sigma_{yy}}{E_b} \\ \frac{\tau}{G_b} \end{bmatrix} \tag{2}$$

Here, E_b, ν_b and G_b are the brick elastic modulus, Poisson's ration and shear modulus, respectively.

3. Two simple homogenization models

In case of linear elastic bricks and mortar joints reduced to interfaces with either linear or non-linear (total strain or holonomic) behavior and within the FE discretization shown in Fig. 1, two simple models are derived and hereafter briefly described. Both compatible homogenization strategies are implemented in