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An improved moving Kriging meshfree method for plate analysis using a refined plate theory

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ABSTRACT

This paper proposes a simple and efficient approach based on a moving Kriging interpolation-based (MKI) meshfree method and a two-variable refined plate theory for static, free vibration and buckling analyses of isotropic plates. A generalized formulation through various higher-order distributed functions is presented. Shear correction factors are not required due to zero-shear stresses satisfied at the top and bottom surfaces of plates. The governing partial differential equations are discretized by a weak Galerkin form and numerically solved by using MKI basis functions. The present theory considers the transverse, shear deflections and their derivations while only the deflections are included in approximate solutions. A new correlation function is proposed to construct MKI shape functions so that the underlying solution becomes stable. In addition, a rotation-free technique based on isogeometric analysis is presented to enforce boundary conditions of normal slopes for clamped plate cases, which is simpler and more efficient than several existing approaches. Numerical results show excellent performance of the present method.

1. Introduction

The plate structures have been received much interest of researchers and various applications into construction, aerospace, mechanical engineering and so on [1] are found. The governing partial differential equations of the present plate theory are a fourth-order and hence a weak form formulation is required the C^1-continuity in approximation fields. In the finite element method (FEM), some C^1-continuous plate elements have proposed [2–4]. However, the implementation of these elements into packages exists some disadvantages related to the mesh geometry. As an alternative approach, the meshless method can be recommended. The meshless method can well capture the higher-order continuity of the present plate theory. Krysl and Belytschko first employed the element free Galerkin (EFG) method for the thin plate static analysis [5]. The EFG was then extended to free vibration analysis [6]. The meshless local Petrov–Galerkin (MLPG) method was applied to bending plate problems [7]. The meshfree particle method was also used for analyzing of the thin plates by Oh et al. [8]. Various researches such as the reproducing kernel particle method proposed by Liu et al. [9], the Hermite-type technique embedded with the reproducing kernel function [10] and the radial point interpolation function (RPIM) [11,12] were also proposed for thin plate analysis. Also, the RPIM incorporated with some smoothing techniques to solve the thin plate problems beyond rotation degrees of freedom was reported in [13]. Further developments of meshfree method for thin shell analyses are found in [14–16] and for fluid–structure interaction problem [17]. In addition, an advanced FE technique based on several versions of strain smoothing was addressed for elasticity [18–22] and plate problems [23,24]. Moreover, isogeometric analysis proposed by Hughes et al. [25] opens a new door for computational mechanics. An overview and computer implementation aspects was recently addressed in [26]. Thanks to the isogeometric analysis, a unified approach for laminated plate structures has been well established [27–29].

The classical plate theory (CPT) is suitable for the thin plate based on Kirchoff-Love assumptions. However, when the plate becomes moderate, the result is no longer accurate by the significant influence of shear deformation. The first-order shear

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deformation theory (FSDT) was then investigated in [30,31]. The FSDT is however required shear correction factors (SCFs) in order to describe properly the shear stress across the plate thickness. The findings of such SCFs depend on many factors: cross-section area, material, etc. [32]. Various higher-order shear deformation theories have been devised to model precisely the stress/strain across the plate thickness. For instance, we can list some higher-order shear deformation theories as the third-order shear deformation theory (TSDT) [33,34], the exponential shear deformation theory (ESDT) [35], the inverse trigonometric shear deformation theory (ITSDT) [36], the fifth-order shear deformation theory (FiSDT) [37], the zigzag model theory [38] and the refined plate theory (RPT) [39]. According to the RPT, the transverse displacement is regarded as a contribution of bending w^b and shear w^s components. Further developments of the RPT can be found in Shimpi et al. [40,41] and Thai et al. [42,43]. Its main advantages are less variable, applicable to thick and thin plate analyses [41]. However, the RPT requires the C^1-continuity in an approximate formulation which the standard finite element does not satisfy. As a suitable method, the MKI function is used to construct the plate formulation based on the RPT.

In attempts to developments of advanced methods, we consider Kriging interpolation initially proposed to evaluate natural resources by Krige [44] and then used to construct the shape functions in meshfree methods. The moving Kriging interpolation shape function was firstly applied to the meshfree method by Gu [45]. Being different from several other meshfree methods, the MKI meshfree method ensures kronecker-delta property of the shape function. Hence, the essential boundary condition is imposed similar to FEM. Thus, the MKI meshfree method procedure is identical to FEM without using boundary correction techniques as Lagrange multipliers [46,47], penalty methods [48,49] and coupling with FEM [50–52]. The MKI shape function is suitable for both the global weak-form and the local weak-form. In the first approach, it was applied to two-dimensional elasticity problems by Tongsuk and Kanok-Nukulchai [53], Sayakoummane and Kanok-Nukulchai [54] then extended the Kriging-based element-free Galerkin method to the degenerated shell structures. Other applications of the moving Kriging interpolation-based meshfree method are of the thin plates [55], the Reissner–Mindlin plates [56]. On the other hand, the local weak-form moving Kriging meshfree method was developed for two-dimensional structural analysis by Lam et al. [57]. Its further developments can be found in [58–60].

In this paper, we present a global moving Kriging interpolation-based meshfree method for static, free vibration and buckling analyses of isotropic plates. Generally, the results from the global MKI meshfree method depend strongly on the quality of the moving Kriging interpolation that is affected by a correlation parameter in correlation function. We therefore propose a correlation function in which the correlation parameter does not influence on the quality of MKI shape function. The present method is proved to be stable for numerical calculations. Several benchmark problems are given to show high efficiency of the proposed method.

The outline of the paper is organized as follows. In Section 2, a brief on moving Kriging interpolation-based meshfree method is introduced. In Section 3, a rotation-free moving Kriging interpolation formulation for refined plate model is presented. Section 4 shows a simple technique to impose essential boundary conditions. Numerical results are provided in Section 5. Section 6 summarizes some concluding remarks.

2. On an improved MKI Meshfree method

Let Ω be the domain in $\mathbb{R}^2$ with the boundary Γ as shown in Fig. 1. Let the nodes be defined by x_I ($I = 1, 2, \ldots, N$), in which N is

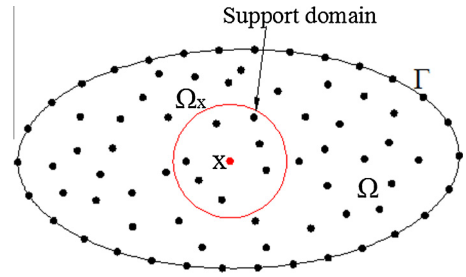

Fig. 1. Domain representation and support domain of 2D model.

the quantity of the nodes in the domain Ω and on the boundary Γ. Consider a support domain $\Omega_x \in \Omega$, the neighborhood of an interested point x located in Ω. In order to approximate u-distributed function in Ω_x that includes a number of nodes x_J ($J = 1, 2, \ldots, n$), where n is the quantity of nodes in the support domain Ω_x.

The moving Kriging interpolation $\mathbf{u}^h(\mathbf{x})$, $\forall \mathbf{x} \in \Omega_x$ can be expressed by

$$\mathbf{u}^h(\mathbf{x}) = [\mathbf{p}^T(\mathbf{x})\mathbf{A} + \mathbf{r}^T(\mathbf{x})\mathbf{B}]\mathbf{u}(\mathbf{x}) \tag{1}$$

The Eq. (1) can be rewritten in a compact form by

$$\mathbf{u}^h(\mathbf{x}) = \sum_{I=1}^{n} \phi_I(\mathbf{x}) u_I \tag{2}$$

The moving Kriging interpolation shape function $\phi_I(\mathbf{x})$ is expressed as follows

$$\phi_I(\mathbf{x}) = \sum_{j=1}^{m} p_j(\mathbf{x}) A_{jI} + \sum_{k=1}^{n} r_k(\mathbf{x}) B_{kI} \tag{3}$$

in which the matrices $\mathbf{A}$ and $\mathbf{B}$ depend on the coordinate of nodes within the support domain Ω_x as

$$\mathbf{A} = \left(\mathbf{P}^T\mathbf{R}^{-1}\mathbf{P}\right)^{-1}\mathbf{P}^T\mathbf{R}^{-1} \text{ and } \mathbf{B} = \mathbf{R}^{-1}(\mathbf{I} - \mathbf{PA}) \tag{4}$$

where vector $\mathbf{p}(\mathbf{x})$ is a polynomial basis of order m and $\mathbf{I}$ is a unit matrix of a size $n \times n$.

For example, the basis functions for a two-dimensional problem can be expressed by a linear and quadratic form, respectively, as

$$\begin{aligned} &\mathbf{p}^T(\mathbf{x}) = \{1 \quad x \quad y\}, \ (m = 3) \text{ and} \\ &\mathbf{p}^T(\mathbf{x}) = \{1 \quad x \quad y \quad x^2 \quad xy \quad y^2\}, \ (m = 6) \end{aligned} \tag{5}$$

In Eq. (4), $\mathbf{P}$ is the matrix of size $n \times m$, which contains the values of all polynomial basis functions at n nodes in Ω_x, as expressed in Eq. (6).

$$\mathbf{P} = \begin{bmatrix} p_1(\mathbf{x}_1) & \ldots & p_m(\mathbf{x}_1) \\ \ldots & \ldots & \ldots \\ p_1(\mathbf{x}_n) & \ldots & p_m(\mathbf{x}_n) \end{bmatrix} \tag{6}$$

and the term $\mathbf{r}(\mathbf{x})$ in Eq. (1) is defined as follows

$$\mathbf{r}(\mathbf{x}) = \{R(\mathbf{x}_1, \mathbf{x}) \quad R(\mathbf{x}_2, \mathbf{x}) \quad \ldots \quad R(\mathbf{x}_n, \mathbf{x})\}^T \tag{7}$$

in which $R(\mathbf{x}_I, \mathbf{x}_J)$ represents the correlation function defined by

$$R(\mathbf{x}_I, \mathbf{x}_J) = \frac{1}{2} E\left[\left(u^h(\mathbf{x}_I) - u^h(\mathbf{x}_J)\right)^2\right] \tag{8}$$

where E denotes an expected value of a random function.

The matrix $\mathbf{R}$ is written as in the following form

$$\mathbf{R} = \begin{bmatrix} R(\mathbf{x}_1, \mathbf{x}_1) & \ldots & R(\mathbf{x}_1, \mathbf{x}_n) \\ \ldots & \ldots & \ldots \\ R(\mathbf{x}_n, \mathbf{x}_1) & \ldots & R(\mathbf{x}_n, \mathbf{x}_n) \end{bmatrix} \tag{9}$$