Computers and Structures 177 (2016) 1–11

Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

Efficient optimization of reliability-constrained structural design problems including interval uncertainty

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ARTICLE INFO

Article history:
Received 8 September 2015
Accepted 9 August 2016

Keywords:
Surrogate model
Optimization
Interval uncertainty
Reliability
Composite structures

ABSTRACT

A novel interval uncertainty formulation for exploring the impact of epistemic uncertainty on reliability-constrained design performance is proposed. An adaptive surrogate modeling framework is developed to locate the lowest reliability value within a multi-dimensional interval. This framework is combined with a multi-objective optimizer, where the interval width is considered as an objective. The resulting Pareto front examines how uncertainty reduces performance while maintaining a specified reliability threshold. Two case studies are presented: a cantilever tube under multiple loads and a composite stiffened panel. The proposed framework demonstrates its ability to resolve the Pareto front in an efficient manner.

1. Introduction

Managing uncertainty, especially in the early stages of structural design, is critical when working with novel designs and materials. While robust design and reliability-based optimization frameworks have been successfully developed (see e.g. [1–4] for recent work in this field), most of these formulations require a precise stochastic definition of the uncertainty involved. However, early stage structural design with novel materials or applications is marked by comparatively limited and vague information about the design and associated uncertainties. Uncertainty associated with limited knowledge is epistemic in nature and is normally reducible by investment in engineering investigation. Consequently, traditional reliability-based design tools may not be well suited to model the design situation as the epistemic uncertainty normally cannot be stochastically defined.

Reliability analysis with non-probabilistic interval uncertainty [5,6] has been studied as an approach to modeling epistemic uncertainty. While this removes the need to define a precise stochastic distribution, reasonable uncertainty bounds still must be developed. Here, an alternative approach is proposed. The amount of uncertainty - the width of the interval - is considered an independent variable. An efficient approach for finding the worst-case reliability with varying interval widths via surrogate models built from reliability calculations is developed. Then, the width of the uncertainty intervals are treated as an objective in a multi-objective optimization approach while maintaining consistent reliability levels. The resulting Pareto fronts show the impact of lack of knowledge on design performance and allow engineering design team to prioritize where to invest time in reducing epistemic uncertainty. The core contribution required to make such an approach practical is a novel adaptive surrogate modeling technique for efficient interval reliability analysis. This surrogate approach allows the combination of reliability models, interval uncertainty, and multi-objective optimization to remain computationally feasible.

The central concept of the proposed framework is that interval uncertainty is a useful representation of uncertainty in early-stage design knowledge. Conventional structural optimization approaches [7] often use stochastic form to account for uncertainty in the design. However, in the field of marine structures, such models are difficult to apply early in the design process as precise stochastic uncertainty information does not yet exist. Any error in assumption of distribution can be harmful later in the design [8]. This is especially true for marine structures, where too low early structural weight estimates can cause extensive design rework or in-service structural failures [9]. It is argued that interval uncertainty can be used to address this concern where no assumption of distribution is needed [10]. Among other non-traditional uncertainty models [11], interval uncertainty was selected as in most cases the non-deterministic parameters and variables are only known within intervals [12]. This work is the first to explore adaptive surrogate modeling to reveal the coupling between struc-

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http://dx.doi.org/10.1016/j.compstruc.2016.08.004
0045-7949/

tural optimization with interval uncertainty while meeting reliability-based constraints. The interval uncertainty range will be treated as design variables, and the optimizer will determine feasible design configurations with respect to various uncertainty intervals. The aim is to resolve the trade-off between interval uncertainties and design performance through the optimization, thus unveiling the overall impact of early stage design uncertainty on the design.

A complication in selecting interval uncertainty is that working with intervals in optimization can be computationally expensive. In design optimization involving interval uncertainty, a max-min optimization [13] is often needed. In min-max approaches, the optimizer searches for a solution that has the best worst-case performance in the uncertainty interval. In terms of reliability analysis involving interval uncertainty, the worst case of interval analysis needs to satisfy the specified reliability constraint. As reliability simulation itself normally involves a search for most probable point of failure [14], reliability analysis with interval uncertainty becomes a nested optimization in which the worst case performance needs to be located. Considering that such analysis is repeatedly requested in population-based metaheuristic design optimization approaches, the computation can soon become intractable. This work presents a method that is capable in locating the worst case reliability result while remaining computationally efficient.

Many previous authors have studied how to efficiently solve reliability-based design optimization (RBDO) problems. One way is focused on reliability problem formulation, where the performance measure approach [15], sequential RBDO [16] and single-loop RBDO [17] have been proposed. However, they are not ideal as solutions to interval uncertainty problems where worst case reliability needs to be computed. A more direct way to reduce computational cost is to use surrogate models [18]. Among the surrogate methods proposed, Kriging [19] shows promise as an approximation tool in reliability simulation. Kriging was first proposed for structural reliability problem by Kaymaz [20], and recent development can be found in Bichon et al. [21], Echard et al. [22], and Dubourg et al. [23]. These studies on Kriging methods for reliability mainly focused on the approximation of the limit state function, and then applying Monte Carlo Simulation (MCS) method for reliability analysis. Such methodology has also been applied in solving reliability analysis with interval uncertainty problems [24]. While Kriging-assisted MCS methods may be a viable strategy for a single reliability analysis, adopting this methodology in population-based optimization algorithms, such as evolutionary algorithms, can be problematic, as the large number of reliability analyses required can quickly render MCS method extremely costly even with the help of Kriging.

This study introduces two new refinements to the interval uncertainty problem with reliability constraints first presented at the MARSTRUCT 2015 conference [10]. First, a new online surrogate model construction technique is proposed, where the optimizer can dynamically refine an initial coarse surrogate model over the course of the optimization. Second, a quadratic approximation strategy is used to remove the innermost search on the surrogate model - that for worst-case reliability value in a bound interval. By coupling these strategies together, the computational burden of the proposed method is significantly reduced. Less time is spent building the surrogate model upfront. The quadratic approximation, when coupled with sequential refinement of the surrogate has proven reasonable in practice for cases where the worst-case performance is located in the interior of the interval search range. More commonly, the worst-case performance is located at the upper bound of the uncertainty interval, and in this case the quadratic approximation is instead used to determine which interval boundary has the worst performance. The method proposed here can accurately estimate worst-case reliability performance for multi-objective evolutionary algorithm with a high level of efficiency.

The rest of the paper is organized as follows. Section 2 reviews the interval uncertainty and interval reliability analysis. Subsequently, Section 3 introduces the proposed surrogate modeling method and the multi-objective optimization framework for trade-off study. In Section 4, the proposed method is examined in an interval reliability benchmark problem [5], and then the validated method is applied to CFRP top-hat stiffened panel design [25]. Discussion and concluding remarks are given in the last section.

2. Review of interval reliability analysis

This work treats uncertainties that are due to lack of information via an interval formulation. There are two critical components to such an approach: the definition of the interval model and the application of this model in reliability analysis. Each of these components is reviewed in turn in this section.

2.1. Interval uncertainty

Interval uncertainty provides an appropriate alternative from stochastic uncertainty, as no information regarding stochastic distribution is required. Interval modeling [26] is usually applied in a simple closed form:

$$Y = [Y_l, Y_h] = \{Y \in I | Y_l \leqslant Y \leqslant Y_h\} \tag{1}$$

An interval uncertainty model is concerned with investigating the whole range of the potential values bounded by a higher bound and a lower bound. There is no assumption of probabilistic distribution within these bounds; specifically, a uniform distribution is not assumed. The interval definition can also be interpreted in another form:

$$\forall Y \in [Y_0 - \Delta, Y_0 + \Delta] \tag{2}$$

where Δ is defined as the maximum deviation of uncertainty from nominal value, representing the range of the interval uncertainty. In this form, it is clear that interval uncertainty can reflect the tolerance for error in the design [27]. The impact of interval ranges on the design performance is the aim of this work. This trade space can be studied by making a version of Δ one for the objectives of the optimization problem, as reviewed in Section 3.

2.2. Worst-case reliability index with interval uncertainty

Without interval uncertainty, reliability analysis is concerned with calculating the probability of failure in a limit state function:

$$P_f = Pr(g(\mathbf{X}) \leq 0) = \int_{g(\mathbf{X}) \leq 0} f_{\mathbf{X}}(\mathbf{X}) d\mathbf{X} \tag{3}$$

where g denotes the limit state at which the system is safe if $g(\mathbf{X}) \geqslant 0$. $\mathbf{X}$ is a vector of random variables accounting for stochastic uncertainties, and $f_{\mathbf{X}}(\mathbf{X})$ is the joint probability density function. Required and achieved values of P_f are normally given in terms of the safety index, β, with a standard normal distribution Φ:

$$P_f = \Phi(-\beta) \tag{4}$$

As direct integration of the expression given in Eq. (3) is difficult, many approximate methods of estimating β have been proposed. In this work, a surrogate model will eventually be built from the reliability output (e.g. in the β space, not in the $g(\mathbf{X})$ space.) As we will consider closed-form limit states in this work, the first order reliability method (FORM) [14] is used for simplicity of derivation,