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Dynamic compliance optimization: Time vs frequency domain strategies

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A R T I C L E I N F O

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A B S T R A C T

Dynamic compliance (structural and topology) optimization is a topic of active and fertile research carried on by several research groups aiming to extend to the dynamic regime by now well-consolidated approaches for static compliance optimization. Available approaches on this purpose are first divided into time-domain and frequency-domain strategies and then a comparison between the two is performed with respect to actual significance and CPU time of relevant optimal designs. For this paper sake reference is made to the optimal design of viscoelastic thin beams but the approach may be shown to apply with no modifications to other and possibly more complex systems such as 2D and 3D dynamic-compliance topology optimization. By extensive numerical investigations, it is shown that the frequency-domain approach to be preferred over time-domain schemes even though relevant computations happen to be heavier, mainly as far as the computation of the H_∞-norm of the system transfer function is concerned.

1. Introduction

The (size, shape and topology) optimization of structures in dynamic regime dates back some thirty years ago at least and has been addressed using typically frequency or time domain approaches. With no aim of completeness, a brief summary of the main contributions relevant to the present paper are summarized next whereas reference is made e.g. to [1] and references therein for a comprehensive review concerning the optimization of structures subjected to transient loads.

As to frequency domain approaches, in the pioneering paper [2], a frequency domain approach for the topology optimization of structures is derived that is based on the minimization of the dynamic compliance, a concept that to the best knowledge of this author was therein introduced. Under the (strong) hypothesis of a time-varying sinusoidal load (i.e. characterized by a single frequency) either the compliance, i.e. the dot product between the loads and the dual displacements, or its integral average over a finite frequency interval are minimized. Optimal topologies are then found that depend on the specific frequency of the acting load. Kikuchi and coworkers developed their analysis further in [3] where optimal topologies are found that minimize either the dynamic compliance caused by a sinusoidal load or the so called mean-eigenvalue, i.e. a weighted average of specific eigenfrequencies. More recently [4] extends the approach to handle the case of harmonic excitations characterized by multiple frequencies by minimizing the integral of the displacement amplitude in a given frequency interval. To that goal, a non uniform partition of the frequency axis is introduced that is generally coarse but finer and finer in the vicinity of the poles of the system. A similar approach is proposed in [5] that is further coupled to a model-order reduction approach aiming to lessen the computational burden. Along a similar path optimal topologies that minimize the dynamic compliance due to a thermal action are found in [6] whereas Liu et al. [7] faces the subtle issue of minimizing the dynamic compliance in the presence of rotating harmonic loads characterized by a single frequency. Under the same hypothesis on the acting loads, Xu et al. [8] introduces a concurrent design strategy for the optimal design of composite macrostructure and multi-phase material microstructure for minimum dynamic compliance. A strategy for topology optimization of magnetorheological fluid layers in sandwich plates for semi-active vibration control is proposed in [9], where either a single-frequency harmonic load is considered or, to account for the possible variability of the frequency content of the excitation, an objective function is introduced that is the maximum of all possible dynamic compliances computed for each single harmonic load. To summarize with, frequency domain approaches for minimizing the dynamic compliance are being investigated and applied to a wide variety of problems of engineering interest but are mostly limited to the case of single-frequency harmonic loads which should be considered a severe limitation: even though the loads acting on real life structures may be band-limited one may hardly think that a single frequency harmonic load might be representative of the actual conditions wherein the designed specimen is expected to operate.

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As for time domain approaches, nearly all the existing methods define the dynamic compliance to be minimized as a positive definite function that is integrated in the time domain over a finite time horizon. A semi-analytical sensitivity computation is then derived that is based on the adjoint approach as described in [1]. A 1D wave propagation problem is analyzed and solved in [10] that is based on this very same approach, whereas a 3D approach for a similar problem is proposed in [11] wherein the objective is the material microstructure optimization for linear elastodynamics wave management. From a methodological point of view, the fundamental paper [12] is worth mentioning that enters the subtle issue of the consistency of adjoint-based approaches in the two versions of *differentiate-then-discretize* and *discretize-then-differentiate*. Methodological questions are likewise addressed in [13] where the importance of the selected time-marching scheme is enhanced and in [14] where stress constraints are introduced and an ad-hoc approach for their treatment introduced. The crucial issue of coupling this kind of approaches with model-order reduction strategies is faced in [15] using different goal functions that share the fact of being time-domain averaged integrals but differ as to the specific engineering goal to be minimized that can be the dynamic compliance, the mean strain energy or the mean squared displacement of a target degree-of-freedom.

The remainder of the paper is organized as follows. Section 2 presents a time-domain framework and its dual frequency-domain counterpart for the analysis of dynamical systems dependent on a vector of design parameters. The framework is abstract enough to warrant applicability to a wide range of systems including 2D and 3D topology optimization problems even though the focus of this paper shall be on viscoelastic beams discretized via a truly-mixed finite-element scheme. Sections 2.2 and 2.3 develop the two dynamic-compliance minimization problems framed in the frequency domain and in the time domain, respectively. A fundamental issue that is worth mentioning is that the frequency-domain approach, based on the transfer-function H_∞-norm concept, has the merit of exploiting a general frequency content of the incoming excitation with a single-shot strategy, a feature that is neither shared by the proposed time-domain approach nor by any of the existing methods referred to previously. Section 3 presents the results of a few numerical investigations where the results got by using the two approaches are presented and compared whereas Section 4 presents conclusive remarks and need for ongoing and future developments. For completeness sake, equations governing the viscoelastic beam model investigated herein are presented in Appendix A (for an in depth derivation, see [16]).

2. A unified framework for time and frequency domain dynamic compliance optimization

Aim of this section is to derive an abstract framework for time-domain and frequency-domain dynamic compliance optimization. To this goal the system dynamic governing relations are presented first followed by two formulations for dynamic compliance optimization in time and frequency domain, respectively. At this stage, the setting is abstract and no reference is made to any specific structural system and in fact the proposed approach applies to a wide range of dynamical systems, including 2D and 3D dynamic compliance topology optimization. Given this general scenario, for this paper sake the formulations shall be specified and applied to viscoelastic beams discretized by truly-mixed finite elements that have the merit of allowing an in-depth evaluation of the proposed approach keeping the computations simple enough and leaving the focus on the methodology itself.

2.1. System dynamics governing relations

In view of the developments concerning the optimization formulations to be proposed next, dynamical systems investigated herein are given either the descriptor state-space (DSS) formulation

$$\begin{cases} \boldsymbol{E}\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{w} \\ \boldsymbol{z} = \boldsymbol{C}\boldsymbol{x}, \end{cases} \tag{1}$$

or its Laplace-domain transfer-function counterpart (DTF)

$$\boldsymbol{Z} = \boldsymbol{G}(s)\boldsymbol{W}, \quad \text{where}: \ \boldsymbol{G}(s) = \boldsymbol{C}(s\boldsymbol{E} - \boldsymbol{A})^{-1}\boldsymbol{B}, \tag{2}$$

where $\boldsymbol{x}$ and $\boldsymbol{z}$ are the state and output vectors, respectively, $\boldsymbol{A}$ is the structural state matrix and $\boldsymbol{Z}$ the Laplace transforms of $\boldsymbol{z}$. Within the usual framework [17], $\boldsymbol{w}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ use to be the load vector, the (Boolean) matrix distributing the loads to the degrees-of-freedom and the (Boolean) matrix that determines the entries of the output vector $\boldsymbol{z}$ through a linear combination of the state vector components $\boldsymbol{x}$, respectively. A few modifications are needed in order for the output $\boldsymbol{z}$ to coincide with (a new definition of) the dynamic compliance $\mathscr{C}$. First of all $\boldsymbol{C}$ ceases to be a Boolean matrix but incorporates the load themselves, i.e.

$$\boldsymbol{C} = [F_1 \cdots F_i \cdots], \tag{3}$$

where F_i is the load acting on the i-th component of the state vector $\boldsymbol{x}$. The compliance $\mathscr{C}$ then reads

$$\mathscr{C} \equiv \boldsymbol{z} = \boldsymbol{C}\boldsymbol{x} = [F_1 \cdots F_i \cdots] \begin{bmatrix} u_1 \\ \vdots \\ u_i \\ \vdots \end{bmatrix} = \sum_i F_i u_i, \tag{4}$$

where u_i is the displacement of the i-th degree-of-freedom that is forced by F_i. Likewise for $\boldsymbol{B}$ and $\boldsymbol{w}$. First of all one should notice that, even though the abstract framework of Eqs. (1) and (2) is inherently Multi-Input/Multi-Output (MIMO), see e.g. [18] for MIMO optimal design of dynamic structures, dynamic compliance optimization may be inherently framed as a Single-Input/Single-Output (SISO) problem since all the incoming disturbances belong to the same load case and concur to the determination of the single output, i.e. the compliance $\mathscr{C}$. To accommodate this issue one sets $\boldsymbol{w} = 1$ and

$$\boldsymbol{B} = \begin{bmatrix} \vdots \\ F_k \\ \vdots \end{bmatrix}, \tag{5}$$

where F_k is the forcing term of the k-th dynamic equation of motion in $(1)_1$.

2.2. The frequency-domain H_∞-norm based optimization framework

The design parameters, that may be the element densities of topology optimization or the element beam depths in the case of size and/or shape optimization, are grouped in a vector, say $\boldsymbol{p}$, on which the structural matrices $\boldsymbol{E}$ and $\boldsymbol{A}$ depend. Eq. (2) is then rewritten making such dependent explicit, i.e.

$$\boldsymbol{Z} = \boldsymbol{G}(s;\boldsymbol{p})\boldsymbol{W}, \quad \text{where}: \ \boldsymbol{G}(s;\boldsymbol{p}) = \boldsymbol{C}(s\boldsymbol{E}(\boldsymbol{p}) - \boldsymbol{A}(\boldsymbol{p}))^{-1}\boldsymbol{B}. \tag{6}$$

The abstract frequency-domain dynamic compliance problem therefore reads

$$\begin{cases} \min\limits_{\boldsymbol{p}} \ \|\mathscr{C}(\boldsymbol{p})\|_\infty = \|\boldsymbol{G}(i\omega;\boldsymbol{p})\|_\infty & \\ \text{s.t.} & \boldsymbol{G}(i\omega;\boldsymbol{p}) = \boldsymbol{C}(i\omega\boldsymbol{E}(\boldsymbol{p}) - \boldsymbol{A}(\boldsymbol{p}))^{-1}\boldsymbol{B} \\ & V(\boldsymbol{p}) \leqslant V_{\max} \\ & p_{\min} \leqslant \boldsymbol{p} \leqslant p_{\max} \end{cases}, \tag{7}$$