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Node-based free-form optimization method for vibration problems of shell structures

Masatoshi Shimoda [a], Yang Liu [b,*]

[a] Department of Advanced Science and Technology, Toyota Technological Institute, 2-12-1 Hisakata, Tenpaku-Ku, Nagoya 468-8511, Japan
[b] Department of Mechanical Engineering, Sojo University, 4-22-1, Ikeda, Nishi-Ku, Kumamoto 860-0082, Japan

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ABSTRACT

We have previously proposed a numerical node-based parameter-free shape optimization method for designing the optimal free-form surface of shell structures. In this paper, this method is extended to deal with two vibration problems including a vibration eigenvalue maximization problem and a frequency response minimization problem. To avoid the repeated eigenvalue problem when a specified vibration eigenvalue is maximized, we provide two optional approaches, i.e., tracking the specified natural mode or increasing all the repeated eigenvalues. Each vibration problem is formulated as a distributed-parameter shape optimization problem, and the derived shape gradient function is applied to the H^1 gradient method for the shells proposed by the authors, where the shape gradient function is used as a distributed force function to vary the surface. With this method, the optimal and smooth free-form shape including a natural bead pattern can be obtained. Several calculated examples are presented to demonstrate the effectiveness of the proposed method for the free-form design of shell structures involving vibration problems.

1. Introduction

Shell structures are commonly found in nature and in artefacts. Eggs, seashells and insect shells are typical examples in nature, and plastic bottles, automobiles, aircraft, and civil and architectural structures are typical ones of artefacts. Owing to their characteristics of thinness and lightness, shell structures are apt to become the sources of noise and vibration, which not only cause discomfort to users, but also give rise to strength problems such as fatigue life issues. Various techniques have been implemented over the years to isolate vibration sources and transfer paths, to reduce vibration levels and to control vibration modes. In order to solve this dynamic problem in the design of artificial shell structures, it is important to optimize their shapes or curvature distributions, while satisfying various mechanical characteristics. From the standpoint of environmental problems, it is also necessary to make structures lighter in weight. Numerical shape optimization techniques offer a unique way to resolve these problems and meet strict requirements simultaneously.

In focusing on the shape optimization of shell structures, most of the proposed numerical shape optimization methods for designing the shape of shell structures are classified as parametric methods [1–5], in which a shell is parameterized by using parametric surfaces or design elements in advance. These methods are effective for reducing the number of design variables and seldom cause a jagged boundary problem [6]. However, designers need considerable knowledge of and experience with shape parameterization, especially because the shape obtained is strongly influenced by parameterization. On the other hand, we have developed a free-form optimization method for obtaining the optimal natural shell form [7,8]. The method consists of main three parts; (1) theoretical derivation of the shape gradient function based on the adjoint variable method, (2) numerical computation of the shape gradient function, and (3) determination of the optimal shape variation based on the H^1 gradient method for shells stated before. The method is a non-parametric technique that can determine the optimal smooth and natural free-form shape without causing jagged shell structures. Bletzinger et al. also proposed a parameter-free method for the free-form design of shells, using a filtering technique to maintain smoothness [9,10]. Our proposed method was previously applied to a stiffness design problem [7] and shape identification problem of shell structures [8]. The purpose of this paper was to develop the parameter-free shape optimization method to deal with the dynamic problems of shell structures as mentioned above.

* Corresponding author.
E-mail addresses: liu@mec.sojo-u.ac.jp, liuyang60212@gmail.com (Y. Liu).

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Structural dynamic optimization was considered as an efficient way to reduce the vibration, and a number of studies have been carried out to solve dynamic design problems in the past decades. This studies mainly focus on two topics of structural dynamic optimization. One research topic is about the vibration eigenvalue maximization problem [11–13]. Vibration eigenvalues usually represent the dynamic characteristics of structures, especially the lower order natural frequencies are considered as an evaluation measure of the dynamic response. The dynamic response of structures can be substantially reduced by increasing their lower order vibration eigenvalues [11,14]. Another research topic is frequency response minimization problem [15,16]. It reduces the frequency response at a part of interest of the structure or over the whole structure, so the resonance under the excitation force is suppressed and then the local or global dynamic performance of structure is improved [17]. Unfortunately, almost all of the above mentioned studies address optimization of 2D, 3D continua, and there are few studies contribute to the dynamic optimization of shell structures.

This paper describes a solution to both the vibration eigenvalue maximization problem and the frequency response minimization problem of shell structures based on the free-form optimization method for shells. A specified objective function is minimized under volume and state equation constraints. Especially, the repeated eigenvalue problem in the vibration eigenvalue maximization problem is avoided by two approaches. One is the tracking approach, where the natural vibration mode is tracked by using the MAC (Modal Assurance Criterion) value. Another approach is to change the objective and constraint functions for increasing all the eigenvalues with respect to the eigenvectors in the repeated eigenvalue. Both problems are formulated as a distributed-parameter shape optimization problem. The sensitivity function, called a shape gradient function, and the optimality conditions in each problem are theoretically derived using the material derivative method and the adjoint variable method for each approach. The optimal free-form shape is determined by applying the shape gradient function to the H^1 gradient method. In the following sections, the governing equation of the shell structure, the formulation of the problem, the free-form optimization method and the calculated examples will be described.

2. Governing equation of shell structure assembled by infinitesimal flats

As shown in Fig. 1(a) and Eqs. (1)–(3), consider a thin-walled or a shell structure having an initial bounded domain $\Omega \subset \mathbb{R}^3$ (boundary of $\partial\Omega$), mid-area A (boundary of ∂A), side surface S and plate thickness h. It is assumed for simplicity that stress and strain of the shell structure are expressed by superposing the membrane and bending components and by neglecting their coupling effect. The Mindlin-Reissner plate theory is applied concerning plate bending.

$$\Omega = \{(x_1, x_2, x_3) \in \mathbb{R}^3 | (x_1, x_2) \in A \subset \mathbb{R}^2, x_3 \in (-h/2, h/2)\}, \tag{1}$$

$$\Omega = A \times (-h/2, h/2), \tag{2}$$

$$S = \partial A \times (-h/2, h/2). \tag{3}$$

In addition, it is assumed that the mapping of the local coordinate system $(x_1, x_2, 0)$ which gives the position of the mid-area of the plate, to the global coordinate system (X_1, X_2, X_3), i.e., $\Phi : (x_1, x_2, 0) \in \mathbb{R}^3 \mapsto (X_1, X_2, X_3) \in \mathbb{R}^3$, is piecewise smooth. Planar triangular shell elements are used to discretize shell structures in this work. Using the sign convention in Fig. 1(c), the displacement expressed by the local coordinates $\boldsymbol{u} = \{u_i\}_{i=1,2,3}$ is considered by dividing it into the displacement in the in-plane direction $\{u_\alpha\}_{\alpha=1,2}$ and the displacement in the out-of-plane direction u_3. Considering a plane stress condition as shown in Eq. (4), the displacement u_α and u_3 can be expressed as Eqs. (5) and (6), respectively, by using the Mindlin-Reissner plate theory [18].

$$\sigma_{33} = 0, \tag{4}$$

$$u_\alpha(x_1, x_2, x_3) \equiv u_{0\alpha}(x_1, x_2) - x_3\theta_\alpha(x_1, x_2), \tag{5}$$

$$u_3(x_1, x_2, x_3) \equiv w(x_1, x_2), \tag{6}$$

where $\{u_{0\alpha}\}_{\alpha=1,2}, w$ and $\{\theta_\alpha\}_{\alpha=1,2}$ indicate the in-plane displacement, out-of-plane displacement and rotational angle of the mid-area of the plate, respectively. For easier reference, main symbols and variables in this paper are also described in Table 1.

Then, substituting Eqs. (4)–(6) into the variational equation of motion (i.e., weak form) of the three-dimensional linear elastic theory, eliminating ε_{33}, the weak form of natural vibration equation relative to $(\boldsymbol{u}_{0(r)}, w_{(r)}, \boldsymbol{\theta}_{(r)}) \in U$ can be expressed as Eq. (7), and the steady-state forced vibration relative to $(\boldsymbol{u}_0, w, \boldsymbol{\theta}) \in U$ can be expressed as Eq. (8).

$$a((\boldsymbol{u}_{0(r)}, w_{(r)}, \boldsymbol{\theta}_{(r)}), (\bar{\boldsymbol{u}}_0, \bar{w}, \bar{\boldsymbol{\theta}})) = \lambda_{(r)} b((\boldsymbol{u}_{0(r)}, w_{(r)}, \boldsymbol{\theta}_{(r)}), (\bar{\boldsymbol{u}}_0, \bar{w}, \bar{\boldsymbol{\theta}})),$$
$$(\boldsymbol{u}_{0(r)}, w_{(r)}, \boldsymbol{\theta}_{(r)}) \in U, \quad \forall (\bar{\boldsymbol{u}}_0, \bar{w}, \bar{\boldsymbol{\theta}}) \in U, \tag{7}$$

$$a((\boldsymbol{u}_0, w, \boldsymbol{\theta}), (\bar{\boldsymbol{u}}_0, \bar{w}, \bar{\boldsymbol{\theta}})) - \omega^2 b((\boldsymbol{u}_0, w, \boldsymbol{\theta}), (\bar{\boldsymbol{u}}_0, \bar{w}, \bar{\boldsymbol{\theta}}))$$
$$= l(\bar{\boldsymbol{u}}_0, \bar{w}, \bar{\boldsymbol{\theta}}), (\boldsymbol{u}_0, w, \boldsymbol{\theta}) \in U, \quad \forall (\bar{\boldsymbol{u}}_0, \bar{w}, \bar{\boldsymbol{\theta}}) \in U, \tag{8}$$

where $\bar{(\cdot)}$ expresses a variation. $\lambda_{(r)}$ in Eq. (7) indicates the eigenvalue of the rth natural mode and ω in Eq. (8) indicates the excitation frequency. In addition, the bilinear forms $a(\cdot,\cdot), b(\cdot,\cdot)$ and the linear form $l(\cdot)$ are defined respectively in Eqs. (9)–(11).

$$a((\boldsymbol{u}_0, w, \boldsymbol{\theta}), (\bar{\boldsymbol{u}}_0, \bar{w}, \bar{\boldsymbol{\theta}})) = \int_\Omega \{C_{\alpha\beta\gamma\delta}(u_{0\alpha,\beta} - x_3\theta_{\alpha,\beta})(\bar{u}_{0\gamma,\delta} - x_3\bar{\theta}_{\gamma,\delta})$$
$$+ C^S_{\alpha\beta}\gamma_\alpha\bar{\gamma}_\beta\} d\Omega, \tag{9}$$

$$b((\boldsymbol{u}_0, w, \boldsymbol{\theta}), (\bar{\boldsymbol{u}}_0, \bar{w}, \bar{\boldsymbol{\theta}})) = \rho \int_\Omega \{w\bar{w} + (u_{0\alpha} - x_3\theta_\alpha)(\bar{u}_{0\alpha} - x_3\bar{\theta}_\alpha)\} d\Omega, \tag{10}$$

$$l(\bar{\boldsymbol{u}}_0, \bar{w}, \bar{\boldsymbol{\theta}}) = \int_A (f_\alpha \bar{u}_{0\alpha} - m_\alpha\bar{\theta}_\alpha + q\bar{w}) dA$$
$$+ \int_{\partial A_g} (N_\alpha \bar{u}_{0\alpha} ds - M_\alpha\bar{\theta}_\alpha + Q\bar{w}) ds, \tag{11}$$

where $\{C_{\alpha\beta\gamma\delta}\}_{\alpha,\beta,\gamma,\delta=1,2}$ and $\{C^S_{\alpha\beta}\}_{\alpha,\beta=1,2}$ denote an elastic tensor including bending and membrane components, and an elastic tensor with respect to the shearing component, respectively. In addition, $\{\gamma_\alpha \equiv w_{,\alpha} - \theta_\alpha\}_{\alpha=1,2}$ expresses the transverse shear strain tensors and the constants ρ indicates material density.

Moreover, $\boldsymbol{f} = \{f_\alpha\}_{\alpha=1,2}, \boldsymbol{m} = \{m_\alpha\}_{\alpha=1,2}$ and q denote an in-plane load, an out-of-plane moment and an out-of-plane load per unit area applied on A, respectively. $\boldsymbol{N} = \{N_\alpha\}_{\alpha=1,2}, \boldsymbol{M} = \{M_\alpha\}_{\alpha=1,2}$ and Q indicate an in-plane load, a bending moment and a shearing force per unit length applied on ∂A, respectively. In addition, the tensor subscript notation in this paper uses Einstein's summation convention and a partial differential notation for the spatial coordinates $(\cdot)_{,i} = \partial(\cdot)/\partial x_i$. It will be noted that U in Eqs. (7) and (8) are given by the following equation.

$$U = \{(u_{01}, u_{02}, w, \theta_1, \theta_2) \in (H^1(A))^5 | \text{satisfy the given Dirichlet}$$
$$\text{condition on each subboundary}\}, \tag{12}$$

where H^1 is the Sobolev space of order 1.