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High accuracy post-buckling analysis of moderately thick composite plates using an exact finite strip

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ABSTRACT

A high accuracy finite strip for the buckling and post-buckling analysis of moderately thick symmetric cross-ply composite plates is presented in this paper by using first order shear deformation theory (FSDT). The presented method, which is designated by the name Full-analytical Finite Strip Method (F-a FSM), provides an efficient and extremely accurate buckling solution in which the Von-Karman's equilibrium set of equations is solved exactly, and the out-of-plane mode shapes and critical loads are obtained. In the post-buckling stage, the Von-Karman's compatibility equation is solved exactly with the assumption that the deflected form after buckling is a combination of obtained buckling modes (single or multiple mode shapes). The principle of minimum potential energy is invoked to solve for the unknown coefficients in the assumed out-of-plane deflection and rotations functions.

1. Introduction

The application of plates and plate structures has been increased in many branches of mechanics and aerospace engineering. Prismatic plates and plate structures are often employed in situations where they are subjected to in-plane compressive loading. Thus, it is important to predict the buckling and post-buckling behaviour of such structures accurately. As far as the linear buckling and vibration analysis of composite materials are concerned, the finite strip method (FSM) has been extensively used in the context of classical plate theory (CPT) and first-order shear deformation plate theory (FSDT) [1,2].

The post-local-buckling behaviour of elastic plates or plate structures is a geometric non-linear problem. The non-linearity occurs as a result of relatively large out-of-plane deflection, which necessitates the inclusion of non-linear terms in the strain–displacement equations. Many works have been done concerning the geometrically non-linearity response of the structures by using FSM. Early works are those of Graves Smith and Sridharan [3,4] and Hancock [5].

These investigators assume a plate with simply supported ends and subjected to progressive end shortening in order to predict the post-buckling behaviour of the structure. They also consider the post-buckling behaviour of plate structures subjected to uniform or linearly varying end shortening with each component plate of the structure having simply supported ends. The elastic post-buckling response of channel section struts and rectangular box columns are investigated by Graves-Smith and Sridharan. Hancock has used the finite strip method to investigate the post-buckling behaviour of square box and I-section columns. In the finite strip methods developed by the aforementioned authors, in-plane displacement fields are postulated in addition to the out-of-plane displacement field. The lengthwise variations in the displacement fields are trigonometric functions. The crosswise variations in both in-plane and out-of-plane displacement fields are simple polynomial functions. It is noted that the above-mentioned finite strip methods can be categorized as Semi-analytical FSM (S-a FSM).

An energy-based method, referenced to as the semi-energy method by Rhodes and Harvey [6], was first used by Marguerre [7] and has since been used by various researchers. Chou and Rhodes' [8] paper is mostly based on the semi-energy method while containing some useful experimental data. Khong and Rhodes [9] have set up a computationally efficient approach to the post-buckling analysis of prismatic structural members. In this approach, a linear finite strip method, developed for the buckling analysis, based on the Principle of Minimum Potential Energy is employed to find the buckling eigenvector. This eigenvector is then used as the post-buckled deflected shape in a single-term post-buckling analysis based on the Principle of Minimum Potential Energy. The analysis is simplified by the assumption that stresses in the direction perpendicular to loading and shear stresses have negligible effects. This approach can be considered as a lower

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bound method of post-buckling analysis (i.e. the post-buckling stiffness of the structure is underestimated by this approach). The method is applied to plain and stiffened channel sections as well as Z-sections.

Ovesy et al. [10–12] have developed a Semi-energy post-local-buckling FSM in which the out-of-plane displacement of the finite strip is the only displacement which is postulated by a deflected form. The developed Semi-energy FSM (S-e FSM) has been applied to analyse the post-local-buckling behaviour of thin flat plates [10], open channel section [11] and box section struts [12].

Ovesy and Ghannadpour [13–16] have developed a Full-analytical FSM (F-a FSM) based on CPT and classical laminated plate theory (CLPT) in which the Von-Karman's equilibrium equation is solved exactly and thus the buckling mode shapes and loads are obtained with very high accuracy. Then the first obtained mode shape is used in the post-buckling phase and the Von-Karman's compatibility equation is solved exactly and the in-plane displacements are calculated. Ghannadpour and Ovesy [17,18] have extended their exact method to high accuracy non-linear post-buckling behaviour of isotropic plates and plate structures based on CPT by taking into account a combination of the first, second and higher(if required) modes of buckling. They also have solved the Von-Karman's compatibility equation resulted from substituting the combined deflection function, and thus, the post-buckling behaviour has been investigated with high accuracy. The post-buckling behaviour of laminated composite plates has been investigated by Dash and Singh [19]. The higher order shear deformation theory (HSDT) associated with Green–Lagrange non-linear strain–displacement relationship has been used in this paper and the finite element method has been employed to solve the problem.

Kolakowski and Mania [20] have compared the semi-analytical method and the finite element method based on the CLPT which have been used to predict the local post-buckling behaviour of thin-walled composite structures with closed cross-section. Upadhyay and Shukla [21] have considered the HSDT and have used the von-Karman's equilibrium equation with geometrical non-linear assumptions to investigate the post-buckling behaviour of composite and sandwich skew plates. They have employed finite degree double Chebyshev series method to discretize the governing differential equation and boundary condition and the non-linear terms have been linearized using quadratic extrapolation techniques. Post-buckling of rectangular composite plates has been analyzed by Qiao and Chen [22] using spline finite strip method. In this paper, the FSDT has been employed and the application of this method to post-local-buckling of fibre-reinforced plastic composite structures has been illustrated with discrete plate analysis. The governing differential equation, which is obtained from Principle of Minimum Potential Energy, has been solved using Newton–Raphson method.

More recently, Ovesy et al. [23,24] have developed an exact finite strip for investigating the buckling and initial post-buckling behaviour of the moderately thick composite plates and plate structures based on FSDT. They have contributed the first buckling mode in the post-buckling stage and so a single-mode analysis has been done to investigate the initial post-buckling behaviour of laminated moderately thick plates. In this method the Von-Karman's equilibrium set of equations has been solved analytically and the buckling mode shapes have been found with very high accuracy. Ghannadpour et al. [25] have used their new exact finite strip to investigate the post-buckling behaviour of an isotropic plate using the FSDT. They have contributed more than one buckling mode (the first mode) in the post-buckling stage and so a multi-mode analysis has been done to investigate the post-buckling behaviour of moderately thick isotropic plates with very high accuracy.

Ovesy et al. [26] have presented the theoretical developments of a finite strip for the buckling and post-buckling analysis of moderately thick composite plates based of FSDT assumptions. The Von-Karman's equilibrium set of equations has been solved exactly with the assumption that the two loaded ends are simply supported and the other two ends have arbitrary out-of-plane boundary conditions. The analytical solution function of the set of equations has been developed to a more general function which contains all of the solution conditions. The buckling mode shapes obtained from the buckling phase are used as global shape functions for representing the displacement fields in a geometrically non-linear analysis. The Von-Karman's compatibility equation is then solved exactly to obtain the general form of in-plane displacement fields in the post-buckling region. It is noted that an enhanced multi-mode analysis is also carried out in the aforementioned conference paper. The current paper is an updated and revised version of the latter conference paper [26]. The application and scope of the previous paper are strengthened by studying more previous works in the introduction section and considering more critical results.

2. Theoretical developments of the F-a FSM based on FSDT

In this section, the fundamental elements of the theory for the developed finite strip in buckling and post-buckling problems are outlined. A finite strip which is initially perfectly flat is used throughout the theoretical developments of this paper. It is assumed that the strip is constructed from orthotropic plies with symmetric configuration. Moreover, the laminate possesses out-of-plane orthotropy. The out-of-plane boundary conditions of the finite strip are assumed to be simply supported at the loaded ends and arbitrary at the other two edges. It is worth mentioning that the plate is assumed to be moderately thick, thus the FSDT is applied in the remaining of the paper.

2.1. Basic formulation of the problem

The finite strip, which is schematically shown in Fig. 1, is of length L, width b and thickness h. As mentioned earlier, the finite strip is simply supported out-of-plane at both ends, i.e.

$$w = \varphi_y = M_x = 0 \tag{1}$$

It must be noticed that the FSDT is applied, thus

$$\hat{u} = u + z\varphi_x, \quad \hat{\upsilon} = \upsilon + z\varphi_y, \quad \hat{w} = w \tag{2}$$

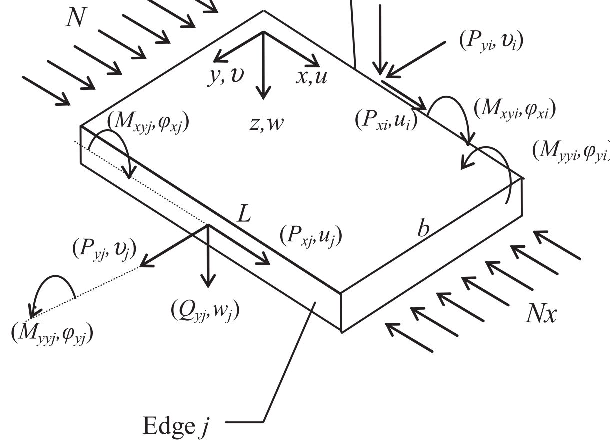

Fig. 1. A typical finite strip.