Computers and Structures xxx (2015) xxx–xxx

Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

Equivalent layered models for functionally graded plates

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ARTICLE INFO

Article history:
Accepted 22 September 2015
Available online xxxx

Keywords:
Functionally graded
Plates
Vibration
Dynamic stiffness
Wittrick–Williams algorithm
Transverse shear

ABSTRACT

Functionally graded plates whose material properties vary continuously through the thickness are modelled as exactly equivalent plates composed of up to four isotropic layers. Separate models are derived for analysis using classical plate theory, first-order and higher-order shear deformation theory. For cases where Poisson's ratio varies through the thickness, the integrations required to obtain the membrane, coupling and out-of-plane stiffness matrices are performed accurately using a series solution. The model is verified by comparison with well converged solutions from approximate models in which the plate is divided into many isotropic layers. Critical buckling loads and undamped natural frequencies are found for a range of illustrative examples.

1. Introduction

Functionally graded (FG) materials can be defined as those which are formed by gradually mixing two or more different materials, with the main aim of adapting their physical properties to the external environment. The variation of properties is required to be as smooth as possible in order to avoid phenomena such as stress concentrations which could lead to the development or propagation of fractures.

Nature provides examples of materials whose physical properties vary gradually, but the concept of synthetically manufactured FG materials was first developed in Japan in the early 1980s [1]. The simplest kind of FG material is made from gradually varying proportions of two constituent materials, usually with complementary properties. For example, in a FG material composed of metal and a ceramic reinforcement, the ceramic material contributes heat and oxidation resistance, while the metal provides toughness, strength and the bonding capability needed in order to minimize residual stresses. Furthermore some crucial properties, such as thermal insulation and impact resistance, can be conveyed to the material by varying the internal pore distribution.

Pioneering manufacturing techniques include powder metallurgy, physical and chemical vapour deposition, plasma spraying, self-propagating high temperature synthesis and galvanoforming [1]. Property changes during FG material processing are commonly performed by functions of the chemical composition, microstructure or atomic order, which depend on the position within the element [2]. Property variation through the thickness of a FG plate is achieved by bulk processing or stacking, layer processing by molecular or mechanical deposition, thermal and electrical preform processing or melt processing. It is also possible to vary properties in the same plane by means of technologies such as ultraviolet irradiation [3]. Jet solidification and laser cladding permit greater variation and are suitable for a wide range of layer thicknesses. Solid freeform fabrication is an advanced production technique which can be controlled by computers.

Although the most important applications of FG materials have taken place in the aerospace industry, mechanical engineering, chemical plants and nuclear energy, they are now attracting attention in optics, sports goods, car components, and particularly in biomaterials by means of prostheses. Modern FG implants allow the bone tissues to penetrate between the metallic (often titanium) part and the bone by means of the hydroxyapatite (a transition porous material), forming a graded layup in which a suitable bonding is developed [1].

In engineering it is important to highlight the effects of FG materials in turbomachinery components such as rotating blades, since by varying the gradation it is possible to alter the natural frequencies in order to guarantee stability at particular spinning speeds. Finally it is worth mentioning smart applications, in which piezoelectric sensors and actuators are integrated into the FG material to control vibrations or static responses in structures [3].

Natural frequencies and critical buckling loads of FG plates have been tabulated by various authors [4–7], and it was shown by Abrate [8,9] that these results are proportional to those for homogeneous isotropic plates. Coupling between in-plane and out-of-plane behaviour can be accounted for by an appropriate choice of

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http://dx.doi.org/10.1016/j.compstruc.2015.09.009
0045-7949/

Please cite this article in press as: Kennedy D et al. Equivalent layered models for functionally graded plates. Comput Struct (2015), http://dx.doi.org/10.1016/j.compstruc.2015.09.009

the neutral surface [10–12]. Thus the behaviour of FG plates can be predicted from that of similar homogeneous plates. These ideas were exploited to obtain an equivalent isotropic model for a FG plate [13] so that it can be analysed using existing methods based on classical plate theory (CPT) for homogeneous plates. This model was shown to give an exact equivalence when the two component materials have the same Poisson's ratio, but otherwise a small approximation is introduced. The analysis of FG plates with varying Poisson's ratio poses a greater challenge due to the complexity of the integrals which have to be evaluated in order to obtain the in-plane, coupling and flexural stiffness matrices, even when the analysis is restricted to CPT. Efraim proposed an alternative approach [14] in which these integrations are approximated, while the present authors performed the integrations accurately [15] using a series solution proposed by Dung and Hoa [16].

The present paper includes the previously derived CPT models and examples [13,15] and then makes important extensions to first-order (FSDT) and higher-order (HSDT) shear deformation theory so as to permit accurate solutions for thick functionally graded plates. Section 2 introduces an equivalent (single layer) isotropic plate model for use with CPT under the assumption that Poisson's ratio does not vary through the thickness of the plate. This assumption is relaxed in Section 3, where an exactly equivalent plate composed of two isotropic layers is derived for CPT by solving an inverse problem to satisfy six independent stiffness requirements. Section 4 demonstrates that the extension of these models to FSDT is trivial, and then outlines extensions to HSDT giving equivalent plates with three and four layers, respectively. The numerical results in Section 5 verify the proposed models, and also demonstrate its accuracy in finding critical buckling loads and natural frequencies of FG plates, using the different plate theories. Section 6 summarises the conclusions and suggests further extensions to the method.

2. Equivalent isotropic plate for CPT with constant Poisson's ratio

Consider a FG plate of thickness h lying in the xy plane with the origin at mid-surface, having material properties which vary through the thickness (z) direction. Using standard notation, the plate constitutive relations of CPT are written as

$$\mathbf{N} = \mathbf{A}\boldsymbol{\varepsilon}_0 + \mathbf{B}\boldsymbol{\kappa} \quad \mathbf{M} = \mathbf{B}\boldsymbol{\varepsilon}_0 + \mathbf{D}\boldsymbol{\kappa} \tag{1}$$

where the vectors $\mathbf{N}, \mathbf{M}, \boldsymbol{\varepsilon}_0$ and $\boldsymbol{\kappa}$ contain perturbation membrane forces per unit length, perturbation bending and twisting moments per unit length, perturbation mid-surface membrane strains, and perturbation curvatures and twist, respectively. The membrane, coupling and out-of-plane stiffness matrices are given by

$$\mathbf{A} = \int_{-h/2}^{h/2} E(z)\mathbf{Q}(z)dz \quad \mathbf{B} = \int_{-h/2}^{h/2} E(z)\mathbf{Q}(z)zdz$$
$$\mathbf{D} = \int_{-h/2}^{h/2} E(z)\mathbf{Q}(z)z^2dz \tag{2}$$

respectively, where

$$\mathbf{Q}(z) = \begin{bmatrix} Q_{11}(z) & Q_{12}(z) & 0 \\ Q_{12}(z) & Q_{11}(z) & 0 \\ 0 & 0 & Q_{66}(z) \end{bmatrix} \tag{3}$$

with

$$\left. \begin{array}{l} Q_{11}(z) = \frac{1}{1-\nu(z)^2} \quad Q_{12}(z) = \nu(z)Q_{11}(z) \\ Q_{66}(z) = \frac{1}{2}(Q_{11}(z) - Q_{12}(z)) \end{array} \right\} \tag{4}$$

Young's modulus $E(z)$, Poisson's ratio $\nu(z)$ and density $\rho(z)$ are assumed to vary through the thickness according to the rule of mixtures

$$E(z) = E_m + V(z)E_\delta \quad \nu(z) = \nu_m + V(z)\nu_\delta \quad \rho(z) = \rho_m + V(z)\rho_\delta \tag{5}$$

where

$$E_\delta = E_r - E_m \quad \nu_\delta = \nu_r - \nu_m \quad \rho_\delta = \rho_r - \rho_m \tag{6}$$

Here, subscripts m and r denote the properties of the metal and reinforcement components, respectively, and $V(z)$ is a function representing the volume fraction of the reinforcement, which is assumed to follow the commonly encountered power law

$$V(z) = \left(\frac{1}{2} + \frac{z}{h}\right)^n \tag{7}$$

The non-negative volume fraction index n controls the variation of the properties of the FG plate, as illustrated in Fig. 1. As n approaches zero the plate consists essentially of reinforcement material, while as n approaches infinity it consists essentially of matrix material.

If both materials have the same Poisson's ratio $\nu_m = \nu_r = \nu_0$, then

$$\mathbf{A} = A_F\mathbf{Q}_0 \quad \mathbf{B} = B_F\mathbf{Q}_0 \quad \mathbf{D} = D_F\mathbf{Q}_0 \tag{8}$$

where

$$\mathbf{Q}(z) \equiv \mathbf{Q}_0 = \frac{1}{1-\nu_0^2}\begin{bmatrix} 1 & \nu_0 & 0 \\ \nu_0 & 1 & 0 \\ 0 & 0 & \frac{1}{2}(1-\nu_0) \end{bmatrix} \tag{9}$$

and

$$\left. \begin{array}{l} A_F = \int_{-h/2}^{h/2} E(z)dz = h\left(E_m + \frac{E_\delta}{n+1}\right) \\ B_F = \int_{-h/2}^{h/2} E(z)zdz = \frac{h^2}{2}\frac{nE_\delta}{(n+1)(n+2)} \\ D_F = \int_{-h/2}^{h/2} E(z)z^2dz = \frac{h^3}{12}\left[E_m + \frac{3(n^2+n+2)E_\delta}{(n+1)(n+2)(n+3)}\right] \end{array} \right\} \tag{10}$$

The presence of B_F indicates coupling between the in-plane and out-of-plane behaviour.

Now consider an isotropic plate of thickness h^*, Young's modulus E^*, Poisson's ratio $\nu^* = \nu_0$, whose neutral surface is offset by δ^* above the geometric mid-surface. The membrane, coupling and out-of-plane stiffness matrices are given by

$$\mathbf{A} = E^*h^*\mathbf{Q}_0 \quad \mathbf{B} = E^*h^*\delta^*\mathbf{Q}_0 \quad \mathbf{D} = E^*\left(\frac{h^{*3}}{12} + h^*\delta^{*2}\right)\mathbf{Q}_0 \tag{11}$$

and is therefore equivalent to the FG plate of Eq. (8) if

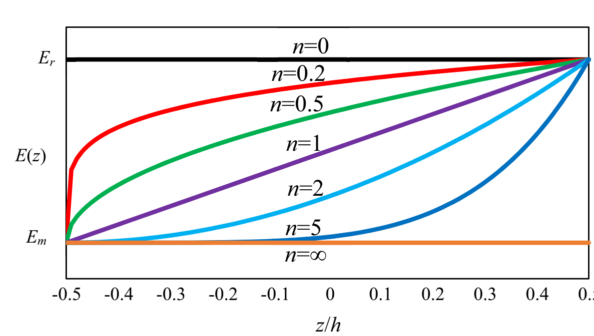

Fig. 1. Variation of Young's modulus through the thickness of a FG plate with volume fraction index n.

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