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Stability of evolutionary brittle-tension 2D solids with heterogeneous resistance

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ABSTRACT

Modeling and analysis of masonry continuum through mechanical models that embed some low tensile skill of the masonry and account for its decay process during time is rarely treated in literature. More sophisticated models are to be considered able to produce results in major agreement with real data.

The development of theoretical formulations aimed at a priori producing evaluations about the overall performance of these models have a central importance, since they allow to make reasoned computational choices about the mechanical models to be referred to for computations.

In this paper one focuses on a special mechanical model for masonry bodies, which is referred to as Elastic Brittle tension (EB) model.

Actually the EB model is evolutionary since the non-null tensile stress yield value is assumed to decay during time and converge towards the No-Tension behavior.

This involves the need of investigating the relationships of the solution with other solutions related to more known mechanical models, which requires for the EB model the development of a proper theoretical formulation that is presented in the paper.

One starts from the consideration that in this case, since the failure in tension is brittle, the theorems of Limit Analysis (LA) are not justified. Thereafter one sets up an approach to the problem aimed at investigating how far the collapse behavior of EB structures can be analyzed through the usual LA tools; some bounding thresholds for their ultimate load-carrying capacity with some original stability statements are then formalized.

The proposed approach is here referred to masonry arches modeled under the EB hypothesis but it may be easily generalized to different structural typologies.

1. Introduction

Most often structural safety of masonry arches is assessed within the context of Limit Analysis. A kinematical approach is adopted, based on the concept of the activation of unilateral hinges, strongly justified by Heyman in his well known paper dating back to 1966 [1].

Later on, the method has been formalized within the theory of No-Tension (NT) structures, and the two classical theorems (static and kinematical) of general Limit Analysis have been demonstrated to hold even for NT materials under proper assumptions (see e.g. [2,3]).

In this case, geometrical models are usually adopted composed by a number of stones linked to each other by a mortar matrix in the case of conglomerate masonry, or, at the opposite extreme, by a number of stone ashlars connected by mortar joints; both the schemes allow for the hypothesis of formation of unilateral hinges.

Collapse mechanisms are then activated when a sufficient number of unilateral hinges is formed at suitable locations.

Anyway, in masonry structures a certain tensile strength can be always recognized in the material, which is usually ignored in mechanical models that are usually adopted in research papers, such as No Tension models.

Although this appears a relevant feature because of the chance of providing more realistic models, a lack of theoretical research may be recorded with reference to this research stream, since few models exhibiting some tensile resistance are available up to now.

From the first studies on the topic [1,4,5], a wide scientific literature witnesses the big research effort produced during the last decades in the field of analysis of masonry constructions. It addresses a number of still open problems and gives the general

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framework of the research area [6–19]. This also includes possible applications for the analysis of masonry structures in case of reinforcements by means of new technologies [20–24], which is also of central interest within the field of protection and control of existing structures [25–29].

2. Motivation of the paper

As mentioned, very seldom scientific papers in literature take into account or formulate mechanical models for masonry that embed a certain (even if limited) tensile skill and its change in time, involving a decay process.

On the contrary, accounting for some low tensile strength exhibited by the masonry material, would make the model in major agreement with the real behavior of masonry. In this case, since the NT behavior would no longer apply, there would be the need of providing a different and ad-hoc conceived theoretical treatment and set up.

Actually the masonry tensile strength skill is mostly low, unreliable, and quickly decaying with time, so that one is pushed to neglect it. From the theoretical point of view the very important issue is that the behavior in tension is definitely not ductile: failure in tension is brittle, so that theorems of Limit Analysis (LA) cannot be justified.

The evaluation of the ultimate load carrying capacity of structures based on a Elastic Brittle tension (EB) assumption of the material, thus, needs some deepening.

On the counterpart, the adoption of this model usually involve a major sophistication and complexity in the implementation of the model itself in calculus codes, and an overall highly increased computational effort. Whence final reliability may be disputable, although the model is more realistic.

Under this perspective, the development of theoretical formulations aimed at a priori produce evaluations about the overall performance of more sophisticated models, such as the EB model, have a central importance, since they allow to make reasoned computational choices about the mechanical models to be referred to for computations and numerical investigations.

Within this framework, the paper aims at investigating how far the collapse behavior of EB structures can be analyzed through the usual tools of LA and at identifying and formalizing some new bounding thresholds for their ultimate load-carrying capacity whence some stability statements are inferred.

The theoretical developments reported in the following may be referred (or not) to structures exhibiting uni-axial stress states, such as (but not only) arched structures, where some approximations on the stress fields may be admitted with regards to the material admissibility, thus allowing the uni-axial simplification.

One may demonstrate that such structures made by materials exhibiting different properties in tension (i.e.: elastic, plastic and brittle) show some energetic order properties of the relevant solutions with comparison to the NT solution.

About the relationships between the involved solutions one can state that: (i) the elastic solution yields the minimum of energy, (ii) the ductile solution is smaller than the brittle one, (iii) the brittle solution with the same tensile limit is larger than the ductile one, and (iv) all of them are smaller than the NT solution.

One can finally infer that the NT model yields an approximation of the brittle model endowed with some tensile strength, and the result gets closer and closer as the tensile strength gets smaller.

On the contrary no investigation is available nowadays about the ultimate load carrying capacity of tensioned brittle structures, which is not at all a trivial topic.

Actually, the collapse mechanisms that are admissible for NT structures still hold for EB structures as well; on the other side, collapse may be moved if the tensile strength is null, but it may happen that maybe loads are not sufficient if some tensile capacity, no matter if brittle or ductile, does exist.

Moreover, it is trivial that collapse must occur if no admissible stress field exists able to equilibrate the applied loads without exceeding the maximum tensile strength. If, by contrast, such a field exists, the static theorem of LA ensures that collapse cannot occur in case of ductile behavior, but this is not true if referred to the brittle structure.

Hence it is possible that a EB structure collapses even if the associate ductile structure does not collapse, while on the other side a EB structure cannot collapse if the associate NT structure does not collapse.

In the following these concepts are handled through the analytical treatment in energetic terms and the outcomes are formalized through the enouncement of the stability statements for the EB solution.

3. Modeling hypotheses and conditions for collapse occurrence

3.1. The Elastic-Brittle low tension (EB) material

Altough it is temporary because of the decay with time and brittle, some low tensile resistance is exhibited by the masonry and this feature might be embedded in a more realistic mechanical model.

As mentioned in the above, whilst in general the ductility of the masonry is poorly reliable, a ductile behavior is exhibited by masonry when, once attained the tensile yield threshold, the material experiences indefinite deformations under constant stress, which is to say without any loss of resistance capacity.

Under the NT hypothesis (Fig. 1a), equilibrium against external loads is satisfied by stress fields $\boldsymbol{\sigma}$ of pure compression (i.e. stress fields required to be negative semi-definite, and, in case, of uni-axial stress states simply non positive); compatibility of strains $\boldsymbol{\varepsilon}$ is ensured by the superposition to the elastic field ε_e of a fracture field ε_f that does not admit any contraction at any point and along any direction within the body (i.e. a fracture field required to be positive semi-definite). Stability of the material in the Drucker's sense is intrinsic to the model.

NT equilibrium is ensured, provided that the loads are under the collapse threshold.

The formulation of the EB behavior model may represent an improvement of the NT modeling since it should provide a model whose behavior is closer to the real one.

The low tension model may be, then, formulated by adopting a non-null tensile resistance with a brittle behavior in tension as shown in Fig. 1b for a uni-axial stress–strain process.

After denoting by "τ" the parameter governing the loading process (for example the time variable), and, for any value "t" of "τ", one gets

$$\sigma(t) = \begin{cases} E\varepsilon(t) & \text{if} \quad \begin{cases} \varepsilon(t) \leqslant 0 \\ \text{or} \\ \max\limits_{0 \leqslant \tau \leqslant t} \varepsilon(\tau) \leqslant \varepsilon'_o \end{cases} \\ \\ 0 & \text{if} \quad \begin{cases} \varepsilon(t) \geqslant 0 \\ \text{and} \\ \max\limits_{0 \leqslant \tau \leqslant t} \varepsilon(\tau) > \varepsilon'_o \end{cases} \end{cases} \tag{1}$$

where σ and ε represent the uni-axial stress and strain states respectively, ε'_o denotes the positive limit strain value, and E the Young's modulus.

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