Engineering Analysis with Boundary Elements 85 (2017) 30–42

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

Numerical simulation of crack growth in piezoelectric structures by BEM

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ARTICLE INFO

Keywords:
Mechanical energy release rate
Hoop stress intensity factor
Piezoelectric
BEM
Interaction
Crack propagation

ABSTRACT

In this paper, a dual boundary element computer program is developed for numerical simulation of a crack propagating in piezoelectric plates under a combined quasi-static electric and mechanical loading. To determine the crack growth path, two fracture criteria are taken into account: the maximum of hoop stress intensity factor (HSIF) and hoop mechanical strain energy release rate (HMERR). By using the displacement extrapolation method, these fracture parameters of any small kinked crack branch are obtained and validated by comparing with the available analytical results. The critical fracture loads for some test specimens are numerically analyzed based on the maximum HMERR fracture criterion. Different electrical boundary conditions on the crack faces are considered and checked with the experimental data. Finally, one crack or a pair of cracks propagating in infinite or finite piezoelectric plates is numerically simulated. The influences of the loading conditions, the anisotropic fracture toughness and the interaction between the cracks on the crack growth paths are also studied. The comparisons with the exiting finite element results show the accuracy and efficiency of the present BEM program for numerical simulation of crack growth in piezoelectric materials.

1. Introduction

The fracture behavior of piezoelectric material has been widely discussed in recent years, but most existing works are confined to stationary cracks. To assure a sufficient reliability of intelligent devices for advanced engineering applications, a thorough understanding of the possible crack initiation and a quantitative prediction of the crack growth are imperative.

The cracks embedding in the piezoelectric matrix essentially can be described as the dielectric interfaces between the solid bodies and the crack medium like fluids or vacuum. But no sufficient experimental data are available to clarify the detailed boundary conditions at the dielectric interfaces. To date, several crack models have been proposed by considering different hypothetically electrical boundary conditions (BCs) across the crack faces. The permeable crack model proposed by Parton [22] assumed that the electric potentials are equal on both crack faces. The impermeable crack model was proposed by Deeg [4] and Pak [19] by ignoring the permittivity of the crack gap. These two models are physically unconvincing since the effect of the crack medium is entirely neglected. Accordingly, by considering the subsistent permittivity of the crack medium, a more exact electrical BCs was proposed by Hao and Shen [9], termed it as a semi-permeable crack model. The electric field within the crack gap is dependent on the displacement and electric potential jumps across the crack faces. From the mechanical point of view, the crack faces were still assumed to be free of tractions. But McMeeking [18] found a discrepancy in a conservative system between the total energy release rate (ERR) and the local crack-tip ERR. To overcome this, Landis [14] further considered the Coulomb forces induced by the electric charges on the crack faces and presented an energetically consistent crack model. Consequently, an open question would be: is this crack model more adaptable than the others for dealing with the piezoelectric crack problems? This issue is one of the problems that will be considered and examined in this paper.

To predict the crack growth in piezoelectric materials, several fracture criteria have been introduced in the literature. According to the characteristic facture parameters, some of them can be classified into energy-based criteria which related to the total ERR G, the mechanical energy release rate (MERR) G_M [21], the local ERR G_C [6], the energy density S [30], etc. The total ERR G predicts that an electric field always inhibits cracking, which obviously contradicts with the experimental results. The maximum G_M criterion can accurately predict the influence of electric field on cracking, but it does not take into account the electrical term without physical interpretation [8]. Some stress-based criteria are also proposed on basis of a generalized stress intensity factor (SIF) and a crack opening displacement (COD) by Fang et al. [5], a modified hoop SIF $K_{\omega\omega}$ by Xu and Rajapakse [27], etc. By comparison of their calculation results with the experimental data, Fang et al. [5] stated that the calculated critical fracture loads based on the COD, G_M and G_C are in

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https://doi.org/10.1016/j.enganabound.2017.09.015
Received 16 July 2017; Received in revised form 13 September 2017; Accepted 24 September 2017
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good agreement with the experimental results. Lei and Zhang [17] observed that a positive electric field will decrease the maximum $K_{\omega\omega}$, which contradicts some existing experiments. It should be stated that suitable fracture criterion particularly for piezoelectrics are still under development.

In general, the maximum or minimum value of the fracture parameters should be accurately computed to determine the crack extension direction, regardless of what kind of fracture criterion is adopted. Most of those fracture parameters are related to the crack-tip field intensity factors either directly or indirectly. Then the key problem turns to how to accurately calculate the generalized SIFs $\{K_\mathrm{I}, K_\mathrm{II}, K_\mathrm{D}\}$ for a vanishingly small kink tip at any kinking angle. By modeling the kink as continuously distributed edge dislocations, the dislocation method [1,27,29] is a powerful way for studying crack kinking problems. But it is confined to solve straight crack problems with a small kink in an infinite medium. Additionally, this is still a time-consuming work to find the maximum value by searching any angle around the crack tip. To overcome this, some alternative parameters were proposed which only depend on the current main crack before the next propagation. Lei and Zhang [17] defined a hoop mechanical energy release rate (HMERR) G_M^{ω} for piezoelectric solids. By using the dislocation method, Xu and Rajapakse [27] compared the generalized hoop SIFs $\mathbf{K}_{\omega}^{\mathrm{m}}(K_{\omega\omega}, K_{r\omega}, K_{D\omega})$ of a main crack with $\mathbf{K}^{\mathrm{b}}(K_\mathrm{I}^\mathrm{b}, K_\mathrm{II}^\mathrm{b}, K_\mathrm{D}^\mathrm{b})$ of a vanishingly small kink for different poling direction and loading conditions. It was found that the differences between them were within 1.0% for a remarkably wide range of kink angle under pure mechanical or electric loading. Lei and Zhang [17] further studied this by virtue of the dual BEM and found that the differences between the maximal SIF $K_\mathrm{I}^\mathrm{b}(\omega)$ and MERR $G_\mathrm{M}^\mathrm{b}(\omega)$ of a small kinked branch, with the maximum values of HSIF $K_{\omega\omega}^\mathrm{m}$ and HMERR $G_{\mathrm{M}\omega}^\mathrm{m}$ of the main crack, are almost within a range less than 5% for various loading cases, which may be neglected for practical applications. It is verified that the proposed HMERR $G_{\mathrm{M}\omega}^\mathrm{m}$ is a good approximation for the crack-tip MERR $G_\mathrm{M}^\mathrm{b}(\omega)$ for a sufficiently small kinked crack branch, which is more effective for the numerical simulation of the crack propagation in piezoelectric materials.

Although numerous scientific works exist on the stationary crack analyses in piezoelectric structures, only a few attempts through numerical methods have been made to simulate crack propagation. The finite element method is the mostly exploited methods in Kumar and Singh [11–13]. Jański and Kuna [10] simulated the stationary and propagating cracks in piezoelectric test specimens with a self-developed adaptive finite element computer program. Bhattacharya et al. [2] dealt with the fatigue crack growth in piezoelectric material using the extended finite element method (XFEM). The impermeable crack-face boundary condition is considered. Bui [3] also numerically simulated the quasi-static crack propagation in piezoelectric solids using the extended isogeometric analysis (XIGA). The maximum modified hoop stress intensity factor criterion was employed for predicting the growing direction of crack. In contrast to the domain discretization, an alternative numerical treatment by BEM offers the advantages of a reduced dimension and an easy representation of arbitrarily curved and moving boundaries. Yet, as far as we know, a few BEM applications have been reported for simulating the crack growth in piezoelectric materials.

In this paper, a computer code written in Fortran program based on the dual boundary element method is developed for simulating the crack growth in piezoelectric plates. The program includes two modules. In the *Fract-Anls* module, the generalized displacement jumps across the crack faces are numerically computed by the dual boundary element method [15,17]. The field intensity factors of any kinked branch-tip are extracted using the displacement extrapolation method and the accuracy of the present BEM results will be examined with the available theoretical results. Then the required fracture parameters will be achieved according to the relationship with the field intensity factors. The critical fracture loads of some test specimens are numerically computed based on the maximum G_M^{ω} criterion and compared with the experimental data for verification. Different crack models are also considered. The *Crack-Ext* module carries out the crack initiation and propagation by the selected fracture criterion. Based on this, the crack propagating in an infinite piezoelectric medium and the three-point blending specimen are numerically simulated by adding new crack-tip elements to the current crack tip. The fracture criteria of maximum $K_{\omega\omega}$ and G_M^{ω} together with the anisotropy of the fracture toughness are considered. Some available FEM results are also presented for our comparison purpose. Additionally, the interaction between a pair of edge cracks on the crack growth paths are also numerically analyzed.

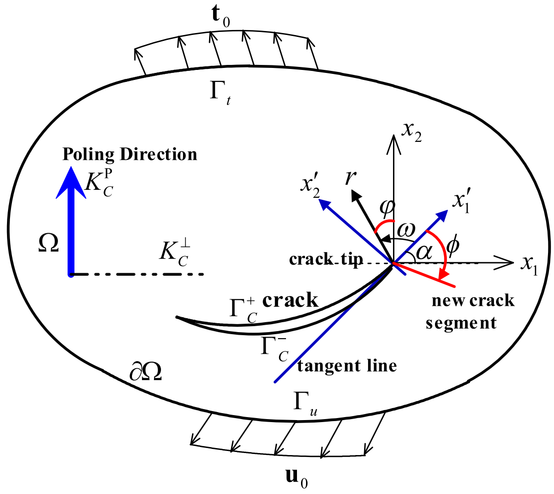

Fig. 1. Configuration and loading: a propagating crack in the piezoelectric plane.

2. Problem statement

2.1. Basic equations for piezoelectric materials

By introducing the generalized displacement vector $\mathbf{u} = \{u_1, u_2, \varphi\}^\mathrm{T}$, strain tensor $\varepsilon = \{\varepsilon_{ij}, -E_i\}$, stress tensor $\sigma = \{\sigma_{ij}, D_i\}$ and elasticity tensor c_{iJKl}

$$c_{iJKl} = \begin{cases} c_{ijkl}, & J, K = 1, 2 \\ e_{jil}, & J = 1, 2; K = 3 \\ e_{kli}, & J = 3; K = 1, 2 \\ -\kappa_{il}, & J, K = 3 \end{cases}, \tag{1}$$

the basic equations for a linear and anisotropic piezoelectric material can be rewritten in a compact form as

$$\sigma_{iJ} = c_{iJKl}\varepsilon_{Kl}, \quad c_{iJKl}u_{K,li} = 0, \tag{2}$$

where σ_{ij}, ε_{ij} and u_i are the mechanical stresses, strains and displacements; E_i, φ and D_i are the electric field, potential and displacements, respectively; c_{ijkl}, e_{kij} and κ_{ik} are the elastic stiffness, piezoelectric and dielectric permitivity constants, respectively. A comma denotes the partial differentiation and the repeated indices imply the conventional summation rule over double indices is applied with all Latin indices. The lowercase and uppercase subscripts take the value 1, 2 and 1, 2, 3; respectively.

2.2. Crack model and boundary conditions

A propagating curved crack embedded in a piezoelectric plate occupying the domain Ω is shown in Fig. 1. A local Cartesian coordinate system (x'_1, x'_2) together with a polar coordinate system (r, ω) are defined at the crack tips. The tangent line along the x'_1-axis is defined in the crack tip as the limit of all tangent lines to the existing crack and approaching the crack tip. The angle α describes the orientation