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Direct time domain evaluation of the transient field transmitted into a lossy ground due to GPR antenna radiation

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ABSTRACT

The paper deals with the direct time domain calculation of a transient electric field generated by a ground penetrating radar (GPR) dipole antenna and transmitted into the lossy half-space. The space–time dependent current along the dipole is governed by the Hallen integral equation which is numerically solved by means of the Galerkin–Bubnov scheme of the Indirect Boundary Element Method (GB-IBEM). Provided that the current distribution along the GPR antenna is determined, one may compute the related electromagnetic field transmitted into the lossy ground by solving the field integral formulas. Some illustrative computational examples related to the transmitted field into the lower half-space are reported in this paper.

1. Introduction

One of the crucial parameter in the study of GPR antenna operation is the transient field transmitted into the lossy half-space. The information of the amount of irradiated electromagnetic energy transferred into the lower medium provides an accurate antenna design and interpretation of the target reflected wave [1–3].

The assessment of the transient field transmitted into the lossy ground can be performed in either frequency or time domain, respectively [4]. Though the most of the direct time domain GPR antenna models are related to the use of the Finite Difference Time Domain (FDTD) method, e.g., [5–7], integral equation approaches, such as the Boundary Element Method (BEM), offer some computational advantages, e.g., avoidance of stair-casing approximation errors [8–9].

Contrary to the widely used FDTD approach, the present paper deals with the assessment of transient electric field transmitted into the ground due to the GPR dipole antenna by means of the Boundary Element Method (BEM).

The direct time-domain analysis of a dipole antenna radiating over a dissipative half-space is based on the space–time Hallen integral equation [8–9], which is numerically solved via the Galerkin–Bubnov variant of the Indirect Boundary Element Method (GB-IBEM) [8–9].

Furthermore, deterministic and deterministic–stochastic model for the transient analysis of GPR dipole antenna radiating above a dielectric half-space is reported in [10,11], respectively.

This paper extends the analysis reported in [10,11] to the more realistic case of a lossy half-space. The analysis of the electric field transmitted into a lower lossy half-space is based on appreciably complex formulation and subsequent more demanding numerical solution procedures than the corresponding analysis of the field transmitted into a lossless medium, which presents significant contribution of this paper. The transient field transmitted into a lossy half-space is evaluated by numerically solving the corresponding field integrals by means of the boundary element formalism. Note that the influence of the air-lossy ground is taken into account via the simplified space–time transmission coefficient arising from the Modified Image Theory (MIT).

Some illustrative numerical results for the field propagating into lower medium are presented in the paper.

2. Theoretical background

The geometry of interest is related to the horizontal straight center-fed thin wire antenna, as shown in Fig. 1. The antenna is positioned at height h above a lossy medium and is excited via voltage source.

2.1. Hallen equation

The transient response of the dipole antenna driven by an equivalent voltage generator is obtained by solving the space–time Hallen integral

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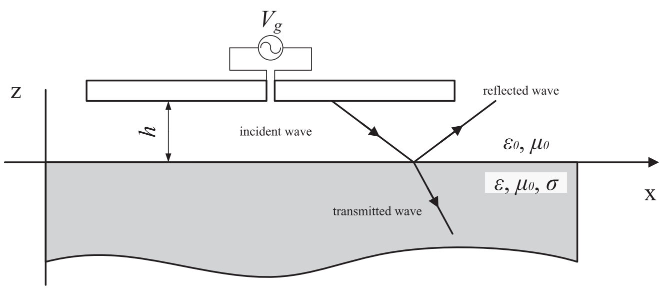

Fig. 1. GPR dipole antenna.

equation for the unknown axial current $I(x,t)$ along the wire [10]

$$\int_0^L \frac{I\left(x',t-\frac{R_a}{c}\right)}{4\pi R_a}dx' - \int_{-\infty}^t\int_0^L r(\theta,\tau)\frac{I\left(x',t-\frac{R_a^*}{c}-\tau\right)}{4\pi R_a^*}dx'd\tau = \frac{1}{2Z_0}V_g\left(x_g,t-\frac{|x-x_g|}{c}\right)+F_0\left(t-\frac{x}{c}\right)+F_L\left(t-\frac{L-x}{c}\right) \quad (1)$$

where c is velocity of light, Z_0 is the free-space wave impedance, while R_a and $R_a{}^*$ denote the distances between observation point x and source point x' on actual and image wire, respectively. Time signals F_0 and F_L account for the multiple reflections from the wire ends, and can be determined by assuming the zero current at the wire ends [8–10]. The interface effects are taken into account via the reflection coefficient (RC) function $r(\theta,t)$ [12]

$$r(\theta,t) = K\delta(t) + \frac{4\beta}{1-\beta^2}\frac{e^{-\alpha t}}{t}\sum_{n=1}^{\infty}(-1)^{n+1}nK^nI_n(\alpha t) \quad (2)$$

where

$$K = \frac{1-\beta}{1+\beta};\ \beta = \frac{\sqrt{\varepsilon_r - \sin^2\theta}}{\varepsilon_r\cos\theta};\ \alpha = \frac{\sigma}{2\varepsilon};\ \theta = arctg\frac{|x-x'|}{2h}, \quad (3)$$

where $\delta(t)$ is the Dirac impulse, and $I_n(t)$ is the nth order modified Bessel function of the first kind. The Hallen equation (1) is solved via space–time version of GB-IBEM [4,10,11].

2.2. Transmitted field

The transient field transmitted in a lossy half-space is given by Poljak et al. [10]

$$E_x^{tr}(r,t) = \frac{\mu_0}{4\pi}\int_{-\infty}^t\int_0^L \Gamma_{tr}^{MIT}(\tau)\frac{\partial I(x',t-R''/v-\tau)}{\partial t}\frac{e^{-\frac{1}{\tau_g}\frac{R''}{v}}}{R''}dx'd\tau \quad (4)$$

where v is velocity of wave propagation in the lower medium and R'' is the distance from the dipole antenna to the observation point located in the lower medium

$$R'' = \sqrt{(x-x')^2+(z+h)^2} \quad (5)$$

The influence of the two-media interface is taken into account via the simplified space–time transmission coefficient arising from the MIT [11]

$$\Gamma_{tr}(t) = \frac{\tau_3}{\tau_2}\delta(t) + \frac{1}{\tau_2}\left(2-\frac{\tau_3}{\tau_2}\right)e^{-\frac{t}{\tau_2}} \quad (6)$$

where

$$\tau_2 = \frac{\varepsilon_r+1}{\sigma}\varepsilon_0,\ \tau_3 = \frac{2\varepsilon}{\sigma} \quad (7)$$

For convenience, expression for the transmitted field can be written, as follows

$$E_x^{tr}(r,t) = -\frac{\mu}{4\pi}\frac{\partial}{\partial t}\int_{-\infty}^t\int_0^L \Gamma_{tr}(t-\tau)I\left(x',\tau-\frac{R''}{v}\right)\frac{e^{-\frac{1}{\tau_3}\frac{R''}{v}}}{R''}dx'd\tau. \quad (8)$$

Furthermore, it is convenient to write a transmitted field in terms of dielectric part and conductive part

$$E_x^{tr} = E_{xd}^{tr} + E_{xg}^{tr} \quad (9)$$

Inserting (6) into (8), utilizing the Dirac impulse property, one obtains:

$$E_{xd}^{tr}(r,t) = -\frac{\mu_0}{4\pi}\int_0^L \frac{\tau_3}{\tau_2}\frac{\partial}{\partial t}I\left(x',t-\frac{R''}{v}\right)\frac{e^{-\frac{1}{\tau_3}\frac{R''}{v}}}{R''}dx' \quad (10)$$

$$E_{xg}^{tr}(r,t) = -\frac{\mu}{4\pi}\int_{-\infty}^t\int_0^L \frac{\partial}{\partial t}\Gamma_{tr}^{II}(t-\tau)I\left(x',\tau-\frac{R''}{v}\right)\frac{e^{-\frac{1}{\tau_3}\frac{R''}{v}}}{R''}dx'd\tau \quad (11)$$

where Γ_{tr}^{II} is the time dependent part of the transmission coefficient (6)

$$\Gamma_{tr}^{II}(t-\tau) = \frac{1}{\tau_2}\left(2-\frac{\tau_3}{\tau_2}\right)e^{-\frac{t-\tau}{\tau_2}} \quad (12)$$

It should be pointed out that, when performing the analysis of the lossless medium, as in [10], the transmitted field is expressed only with relation (10), which is significantly less demanding formulation thus providing appreciably simple and fast numerical solution procedure.

Furthermore (11) it can be written:

$$E_{xg}^{tr} = -\frac{\mu}{4\pi}\int_{-\infty}^t\int_0^L \Gamma_{tr}^{d}(t-\tau)I\left(x',\tau-\frac{R''}{v}\right)\frac{e^{-\frac{1}{\tau_3}\frac{R''}{v}}}{R''}dx'd\tau \quad (13)$$

where

$$\Gamma_{tr}^{d}(t-\tau) = \frac{\partial}{\partial t}\Gamma_{tr}^{II}(t-\tau) = \frac{\partial}{\partial t}\left[\frac{1}{\tau_2}\left(2-\frac{\tau_3}{\tau_2}\right)e^{-\frac{t-\tau}{\tau_2}}\right] = -\frac{1}{\tau_2^2}\left(2-\frac{\tau_3}{\tau_2}\right)e^{-\frac{t-\tau}{\tau_2}} \quad (14)$$

The field integrals (10) and (11) are handled via the boundary element formalism, presented below.

2.3. Numerical evaluation of the transmitted field

The local expansion for the space–time dependent current can be written, as follows

$$I\left(x',t-\frac{R''}{v}\right) = \sum_{i=1}^{N_g} I_i\left(t-\frac{R''}{v}\right)f_i(x') \quad (15)$$

which can also be written in the matrix notation:

$$I(x',t') = \{f\}^T\{I(t')\}. \quad (16)$$

This paper deals with linear approximation which was shown to be sufficient for straight wire problems [4]. For the case of linear approximation, the spatial shape functions are of the form:

$$f_r(x') = \frac{x_{r+1}-x'}{x_{r+1}-x_r},\ f_{r+1}(x') = \frac{x'-x_r}{x_{r+1}-x_r} \quad (17)$$

where $r = 1, 2$.

The space discretization yields:

$$E_x^{tr} = \sum_{i=1}^{M}\left(E_{xd,i}^{tr} + E_{xg,i}^{tr}\right) \quad (18)$$

where the corresponding field integrals are given by:

$$E_{xd,i}^{tr}(r,t) = \frac{\mu_0}{4\pi}\frac{\tau_3}{\tau_2}\left[\int_{\Delta l_i}\frac{\partial}{\partial t}\{I\}\Big|_{t''=t-\frac{R''}{v}}\{f'\}^T\frac{e^{-\frac{1}{\tau_3}\frac{R''}{v}}}{R''}dx'\right], \quad (19)$$

$$E_{xg,i}^{tr} = -\frac{\mu}{4\pi}\int_{-\infty}^t \Gamma_{tr}^{d}(t-\tau)\sum_{i=1}^{N_g} I_i(\tau)\int_{\Delta x_i}\{f'\}_i^T\frac{e^{-\frac{1}{\tau_3}\frac{R''}{v}}}{R''}dx'd\tau. \quad (20)$$