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Horn effect prediction based on the time domain boundary element method

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ABSTRACT

A time domain boundary element method (TBEM) is applied to predict the horn effect. As the time response calculated by the time domain boundary integral equation contains the resonance components, when transformed to the frequency domain, the result will corrupt at the characteristic frequencies. To overcome this problem, a Burton–Miller-type combined time domain integral equation in half-space is applied. The resonance components are excluded in the time domain calculation, thus the corruptions are avoided in the frequency domain. As a result, the horn effect can be predicted very well at all frequencies. Compared to the frequency domain boundary element method for predicting the horn effect, the TBEM is more efficient due to the lower cost of forming coefficient matrices and solving equations. A numerical simulation is carried out to demonstrate the efficiency of the TBEM, and two experiments are conducted to validate the proposed method in predicting the horn effect. Both numerical and experimental results indicate that the proposed method is reliable and efficient in predicting the horn effect.

1. Introduction

Tire/road noise is one of the most important sources of noise pollution in urban areas. The mechanism of tire/road noise has been long investigated [1–9]. It has been proved that the horn effect, which states that the sound in a horn-like geometry formed by the tire/road interface would be amplified, plays an important role in generating tire/road noise. The horn effect has been deeply studied both numerically and experimentally. Experimental methods are appropriate in exploring the mechanism of the horn effect, however, they do not fit the prediction of the horn effect due to the critical environmental requirement and the high expense. Therefore, many numerical and analytical approaches have been proposed to predict the horn effect. The analytical methods are usually fast and simple, such as the two dimensional cylinder model [8], the three-dimensional sphere model [10], the ray theory [9,11] and the combined model proposed recently [12]. However, drawbacks of the analytical methods are also clear. Analytical methods usually adopt simplified models, ignoring some details which may be important to the horn effect. As a result, they are not very accurate at all the frequencies of interest. Along with the analytical approaches, some numerical methods are also applied in the horn effect investigation, and one of the most frequently used methods is the boundary element method (BEM). BEM can deal with the sophisticated details of the tire model, and thus more precise results can be obtained. In fact, the horn effect amplification factors calculated by the BEM are usually used as a benchmark [9,12–14].

The BEM is a powerful tool in the horn effect investigation, however, improvement can still be made according to the deficiency of the BEM depicted in the following part. The horn effect prediction using BEM is usually carried out in the frequency domain. One of the disadvantages of the frequency domain boundary element method (FBEM) is the well-known non-uniqueness problem, however, this can be overcome by the CHIEF (combined Helmholtz integral equation formulation) or the Burton–Miller method [15–17]. Another disadvantage of the FBEM in predicting the horn effect is its low efficiency. The FBEM has to calculate the horn effect at each frequency of interest one after another, because the fundamental solution used in FBEM is dependent on frequency and all the coefficients in the FBEM have to be recalculated at different frequencies. The frequency dependent property makes the efficiency of the FBEM low when the number of frequencies of interest is large, as is usually the case with the horn effect prediction.

To improve the efficiency, the time domain boundary element method (TBEM) which uses a frequency independent fundamental solution is considered as an alternative to FBEM in the horn effect prediction. In TBEM, the coefficients need to be calculated only once for all the frequencies. In this way, a lot of time can be saved in the calculation. To realize the horn effect prediction using TBEM, the response to a wide-band signal at the receiver position is firstly calculated by solving the time domain boundary integral equation (TDBIE), and then the response is transformed to the frequency domain to get the spectra, at last, the horn effect can be predicted using the spectra. Nezhi [18] investigated the horn effect using the TBEM in his research on the sound radiation

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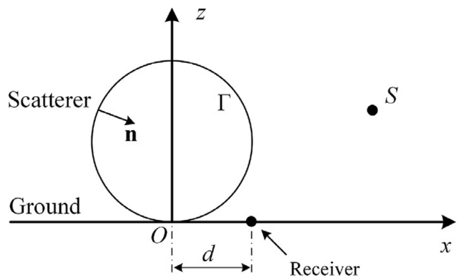

Fig. 1. Geometry of the half-space scattering problem. The scatter is placed on the ground with a smooth boundary Γ, the normal vector **n** to the boundary Γ points inward. A point source denoted as *S* is placed above the ground. The horizontal distance between the origin *O* and the receiver is denoted as *d*.

of tires. However, as a first trial, only a qualitative result was obtained in comparison with the experimental result. In addition, the resonance components of the TBEM [19] which could greatly distort the spectra were not considered in his research.

In this paper, the TBEM is applied to acquire a quantitative and a highly efficient prediction of the horn effect. To ensure a quantitative result, the resonance components of the TBEM are excluded by exploiting a Burton–Miller-type combined time domain integral equation (CTDIE) [19,20] instead of using TDBIE. The use of CTDIE makes the TBEM free from the corruptions at the characteristic frequencies. To illustrate the efficiency of the TBEM in predicting the horn effect, the comparison of the consumed time between the TBEM and the FBEM is also conducted.

2. Theory

2.1. Horn effect prediction

The horn effect problem is actually an acoustic scattering problem in half-space. The schematic geometry of the problem is shown in Fig. 1.

A scatterer representing a tire is placed on the ground. By exploiting the reciprocal theorem which states that the pressures are unchanged when the locations of the source and the receiver are interchanged, a point source S is positioned at a distance from the scatterer above the ground, and the receiver is placed within the tire/road gap. The pressure at the receiver is contributed by four parts: incident pressure from the source and reflected pressure of the source by the ground, and incident pressure from the scatterer and reflected pressure of the scatterer by the ground. The horn effect can be quantified by comparing the total pressure created by the source with the scatterer to the reference pressure created by the source without the scatterer.

The numerical horn effect prediction in the time domain initiates with a wideband transient input, for example, the Dirac impulse. With the source generating an impulse, the response at the receiver can be calculated by solving the TDBIE. Transforming the response to the frequency domain using the fast Fourier transformation (FFT), the amplification factor for describing the horn effect can be obtained through the following formula

$$L_{\text{amp}} = 20\log_{10}\frac{P_{\text{total}}}{P_{\text{ref}}}, \tag{1}$$

where, P_{total} represents the total pressure at the receiver, and P_{ref} represents the reference pressure which can be obtained by transforming the response at the receiver without the scatterer.

2.2. Time domain boundary integral equation

Consider the scattering problem depicted in Fig. 1, the scatterer with a smooth boundary Γ emerges in a homogeneous media with a density of ρ. The unit normal vector at point $\mathbf{x}$ is denoted as $\mathbf{n}_x$, and the normal vector to the surface Γ points inward. The velocity potential at point $\mathbf{x}$ at time t is denoted as $\varphi(\mathbf{x}, t)$. The point source generates an impulse at $t > 0$. Suppose the scatterer is acoustically rigid, the boundary condition, $\partial\varphi(\mathbf{x}, t)/\partial\mathbf{n}_x|_{\mathbf{x}\in\Gamma}=0$, is imposed in this problem. The velocity potentials on the surface of the scatterer can be determined by means of solving the TDBIE which can be expressed as [21]

$$\frac{1}{2}\varphi(\mathbf{x},t) = -\int_\Gamma\int_0^t \frac{\partial}{\partial\mathbf{n}_y}g(r,\tau)\varphi(\mathbf{y},\tau)d\tau d\Gamma + \varphi^i(\mathbf{x},t) \equiv L\{\varphi(\mathbf{x},t)\}, \quad \mathbf{x}\in\Gamma, \tag{2}$$

where L is defined as the velocity potential operator, τ is the retarded time, $r=|\mathbf{x}-\mathbf{y}|$ is the distance between the point $\mathbf{x}$ and the surface point $\mathbf{y}$, $\varphi^i(\mathbf{x},t)$ is the incident item generated by the point source, $g(r, \tau)$ represents the transient fundamental solution in half-space which can be expressed as [22]

$$g(r,\tau) = \frac{1}{4\pi r}\delta\left(t-\frac{r}{c}-\tau\right) + \frac{1}{4\pi r'}\delta\left(t-\frac{r'}{c}-\tau\right) + \sigma, \tag{3}$$

where c is the sound speed, r' is the distance between the surface point $\mathbf{y}$ and the image point of $\mathbf{x}$, δ is the Dirac delta function, and σ is the impedance item. By solving Eq. (2), the surface potentials can be obtained, and then the potential at the receiver can be calculated with a simple substitution of the surface potentials as

$$\varphi(\mathbf{x},t) = -\int_\Gamma\int_0^t \frac{\partial}{\partial\mathbf{n}_y}g(r,\tau)\varphi(\mathbf{y},\tau)d\tau d\Gamma + \varphi^i(\mathbf{x},t), \qquad \mathbf{x}\in\Gamma_+, \mathbf{y}\in\Gamma, \tag{4}$$

where Γ_+ denotes the region outside of the surface Γ. With the potential known, the pressure can be easily derived by the following formula

$$p(\mathbf{x},t) = -\rho\frac{\partial}{\partial t}\varphi(\mathbf{x},t), \tag{5}$$

and then the horn effect of the scatterer can be quantified using Eq. (1).

Although Eq. (2) can be used directly for horn effect prediction, the results are not very accurate at the characteristic frequencies, which are well known for the non-uniqueness problem in the frequency domain. The reason is that the response calculated by the TDBIE contains the resonance components. When the response is transformed to the frequency domain, the corruptions of the TDBIE at the characteristic frequencies can be seen clearly. The corruptions have been discussed in detail in [19], and a Burton–Miller-type CTDIE was proposed to eliminate the corruptions. In the following, the Burton–Miller-type CTDIE is extended to half-space to overcome the corruptions at the characteristic frequencies.

2.3. Burton–Miller-type CTDIE

The Burton–Miller-type CTDIE imposes a linear combination of the time derivative and the normal derivative of the TDBIE, which can be expressed as

$$\frac{1}{2}(1-\alpha)\dot\varphi(\mathbf{x},t) + \frac{1}{2}\alpha c\frac{\partial}{\partial\mathbf{n}_x}\varphi(\mathbf{x},t) = (1-\alpha)L\{\dot\varphi(\mathbf{x},t)\} + \alpha c\frac{\partial}{\partial\mathbf{n}_x}L\{\varphi(\mathbf{x},t)\}, \quad \mathbf{x}\in\Gamma, \tag{6}$$

where α is a real constant ranging from 0 to 1, and

$$L\{\dot\varphi(\mathbf{x},t)\} = -\int_\Gamma\int_0^t \frac{\partial}{\partial\mathbf{n}_y}g(r,\tau)\dot\varphi(\mathbf{y},\tau)d\tau d\Gamma + \dot\varphi^i(\mathbf{x},t), \tag{7}$$

$$\frac{\partial}{\partial\mathbf{n}_x}L\{\varphi(\mathbf{x},t)\} = -\int_\Gamma\int_0^t \frac{\partial^2}{\partial\mathbf{n}_x\partial\mathbf{n}_y}g(r,\tau)\varphi(\mathbf{y},\tau)d\tau d\Gamma + \frac{\partial}{\partial\mathbf{n}_x}\varphi^i(\mathbf{x},t). \tag{8}$$

In Eq. (6), it is obvious that $\partial\varphi(\mathbf{x}, t)/\partial\mathbf{n}_x|_{\mathbf{x}\in\Gamma}=0$, according to the boundary condition. The integral in Eq. (8) contains a hypersingular integral which should be carried out with special care. In this study, a similar procedure to that performed by Terai [23] in the frequency domain is adopted to evaluate the hypersingular integral.