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The extended method of approximate particular solutions to simulate two-dimensional electromagnetic scattering from arbitrary shaped anisotropic objects

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ABSTRACT

The numerical simulation of electromagnetic scattering problem by cylinders of arbitrary cross-section made of homogeneous and anisotropic material is carried out. The anisotropic materials are characterized by symmetric and positive definite magnetic permeability tensors. An efficient and powerful meshfree strong form technique based on the particular solutions of anisotropic radial basis functions (A-RBFs) is extended to deal with anisotropic problem directly. The A-RBFs are constructed by introducing a special norm associated to the appearing anisotropic tensor and replacing it with the standard Euclidean norm in RBFs. The particular solutions for the anisotropic Laplace operator are approximated explicitly by using the anisotropic radial basis functions. Because of material discontinuity at the interface of anisotropic and free regions, two different sets of basis functions, the approximate particular solutions of the anisotropic generalized multiquadric (A-GMQ) and the standard generalized multiquadric (GMQ) functions are employed to approximate the total electric field inside and the scattered field outside the anisotropic scatterer respectively. Some numerical simulations are implemented to confirm the accuracy and efficiency of the method for anisotropic scatterer with arbitrary cross-sections.

1. Introduction

There is a great interest on finding the interaction of electromagnetic (EM) waves and anisotropic objects. This interest is simply because almost all real materials are anisotropic and they are frequently used in optical signal processing, radar cross section (RCS) control, antennas and certain types of radar absorbers. In recent decades, the electromagnetic scattering problems from anisotropic scatterers have been widely studied and analyzed. However, due to tensoral nature of constitutive parameters of anisotropic materials, finding the analytical solution is so difficult and often impossible. So, several types of computational techniques are introduced and extended to deal with these problems. For instance, authors in [1,2] employed the finite difference time domain (FD–TD) method for solving the electromagnetic (EM) waves from anisotropic objects. The frequency domain volume integro-differential equation has been formulated and implemented to deal with the 2-D problems of oblique scattering by penetrable cylinders of arbitrary cross-section made of linear, lossy and anisotropic materials [3,4]. The combined field surface integral equation [5,6] is another approach appropriate when the permittivity and permeability tensors have antisymmetric properties. The finite difference technique with the measured equation of invariance (FD–MEI) has been used to simulate the transversally anisotropic, inhomogeneous cylinder [7,8]. The method discussed in [9] is based on the plane wave representation of the fields in the medium and is only applied to a circular cylinder excited by normal incident or similarly for the case of oblique incidence [10]. Moreover, a hybrid finite element-surface integral method is applied to solve the electromagnetic scattering from a complex inhomogeneous and anisotropic object [11].

There are some difficulties in numerically solving such problem by mesh-based methods like finite difference and finite element methods because of severe anisotropy that requires mesh refinement along the anisotropy direction and consequently the computational cost becomes very expensive. During the last two decades, the meshless methods based on the radial basis functions (RBFs) have been wildly used for solving various types of partial differential equations (PDEs) arising from applied science and engineering [12–22]. Against the mesh-based methods, meshless computational techniques are able to solve PDEs by spreading the nodal points within a geometrical domain without requiring for nodal connectivity. So, they can be just as accurate and are faster, simpler, and need far less data storage. Due to the performance and flexibility of these methods, in recent years they have been successfully employed to investigate some types of practical and complex

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electromagnetic problems [23–33]. One of the effective and stable meshless methods is the method of approximate particular solution (MAPS) based on the radial basis functions [34,35]. In this method the approximate particular solution of the given RBF is used as the basis function in the solution process [36,37]. Recently, the MAPS has been largely used to solve physical and engineering problems such as convection-diffusion problems [38], non-classical 2-D diffusion equation [39], time fractional diffusion equation with a non local boundary [40], Navier–Stokes equations [41] and so on. The MAPS for solving anisotropic elliptic type problems was proposed by Zhu [42].

In the current work, in order to investigate the scattering from homogenous, infinitely long and arbitrarily shaped transversally anisotropic cylinder, a meshless strong form approach based on the approximate particular solution of generalized multiquadrics (GMQ) functions would be implemented. However, the MAPS is not directly applicable to solve intended problem due to the anisotropic operator presented in the respective equation. This difficulty is overcame by using the proper anisotropic radial basis functions (A-RBFs) [43,44]. The A-RBFs are created by replacing the classical Euclidean norm with a new special norm related to the anisotropic operator in the RBFs. In the current work, by using the properties of the new A-RBFs and the particular solutions, an advanced and stable meshless strong form technique would be extended and implemented to investigate the challenging anisotropic scattering problem. Moreover, to deal with material discontinuity at the interface between two different media, two sets of basis functions are considered to approximate the solution in each medium.

This paper is organized as follows: Section 2 presents the mathematical modeling of direct electromagnetic scattering problem from anisotropic objects. The formulation of A-RBFs, MAPS and the RCS calculation will be explained in Section 3. In Section 4 some practical experiments of scattering from infinite anisotropic cylinders with arbitrary cross-sections will be presented to show the accuracy and flexibility of the method.

2. Direct scattering problem from a two-dimensional anisotropic object

Let $D \subseteq \mathbf{R}^2$ denotes the cross-section of an infinitely long anisotropic dielectric cylindrical scatterer located in the free space with outward normal vector n. Moreover, we assume that the scatterer is excited by the $e^{-j\omega t}$ time-harmonic incident plane wave with TM^z polarization where ω is the angular frequency. Let A is a symmetric and positive definite 2 × 2 matrix whose entries are the relative magnetic permeability inside the scatterer. The direct scattering problem for an anisotropic medium is formulated as follows [45]:

$$\nabla\cdot(A\nabla v)+k_0^2\epsilon_r v=0 \quad \text{in } D \tag{2.1}$$

$$\nabla^2 u^s+k_0^2 u^s=0 \quad \text{in } \mathbf{R}^2\setminus\bar{D} \tag{2.2}$$

$$v-u^s=u^i \quad \text{on } \partial D \tag{2.3}$$

$$\frac{\partial v}{\partial n_A}-\frac{\partial u^s}{\partial n}=\frac{\partial u^i}{\partial n} \quad \text{on } \partial D \tag{2.4}$$

$$\lim_{r\to\infty}\sqrt{r}\left(\frac{\partial u^s}{\partial r}-jk_0u^s\right)=0 \quad r=|x|, x=(x,y), \tag{2.5}$$

where v is the scalar total electric field inside the scatterer, u^s and u^i are the scalar scattered and incident electric fields respectively. $k_0=\omega\sqrt{\mu_0\epsilon_0}=\frac{2\pi}{\lambda}$ is the wave number of the free space (λ is the wavelength), where ϵ_0 and μ_0 are the constant electric permittivity and the magnetic permeability of free space. ϵ_r is the relative electric permittivity inside the scatterer. A is of the form

$$A=\frac{1}{\mu_{xx}\mu_{yy}-\mu_{xy}^2}\begin{pmatrix}\mu_{xx} & \mu_{xy}\\ \mu_{xy} & \mu_{yy}\end{pmatrix}$$

and $\frac{\partial v}{\partial n_A}=n\cdot A\nabla v$.

The Sommerfeld radiation condition (2.5) guarantees that the scattered electric field u^s is outgoing. In this work the electric field formulation of the wave scattering problem is studied. The dual magnetic field formulation can be derived equivalently for TE^z polarization by duality $\epsilon\to\mu$ and $\mu\to\epsilon$.

Remark 1. To deal with scattering by finite inhomogeneous anisotropic cylinders, the time harmonic Maxwell's equations in three dimensional (3-D) space are often formulated as volume integral equations. There are different numerical methods that investigated the problem of scattering by 3-D objects. The interested reader for more details refers to [31,46,47].

3. Numerical procedure

Obviously the problem (2.1)–(2.5) is defined in an infinite region. So to solve this problem numerically via the meshless method, the unbounded exterior domain should be restricted to a bounded computational region. For this purpose, suppose that the scatterer is located inside an artificial domain Ω, that is a circle of radius r_a. To have a unique solution, a first order radiation boundary condition (RBC) [48]

$$\frac{\partial u^s}{\partial n}+\left(jk_0+\frac{1}{2r_a}\right)u^s=0, \tag{3.6}$$

is implemented on the circular artificial boundary ∂Ω such that simulates the Sommerfeld boundary condition (2.5). Therefore the direct scattering problem will be converted to

$$\nabla\cdot(A\nabla v)+k_0^2\epsilon_r v=0 \quad \text{in } D \tag{3.7}$$

$$\nabla^2u^s+k_0^2u^s=0 \quad \text{in } \Omega\setminus\overline{D} \tag{3.8}$$

$$v-u^s=u^i \quad \text{on } \partial D \tag{3.9}$$

$$\frac{\partial v}{\partial n_A}-\frac{\partial u^s}{\partial n}=\frac{\partial u^i}{\partial n} \quad \text{on } \partial D \tag{3.10}$$

$$\frac{\partial u^s}{\partial n}+\left(jk_0+\frac{1}{2r_a}\right)u^s=0 \quad \text{on } \partial\Omega. \tag{3.11}$$

Now our interest is to simulate the direct EM scattering problem (3.7)–(3.11) using an advanced computational technique.

3.1. Anisotropic radial basis functions

In recent years, computational meshless methods have been developed and widely used in numerical PDEs. Specially, collocation methods based on the radial basis functions are of the most popular meshless methods. They are truly meshless technique and are very flexible tools to deal with high dimensional practical models with complicated irregular domains. However, the meshless methods based on the classical RBFs are often efficient for isotropic physical models. Authors in [43] showed that the classical RBFs are not proper for directional data problems. In our implementation to directly deal with scattering wave from anisotropic medium and to reduce computational costs, an anisotropic RBF associated to the appearing anisotropic tensor, A, is introduced and employed. For this purpose a special norm called A-norm, associated to the anisotropic tensor, A, in Helmholtz Eq. (3.7) is introduced as follows [42]:

$$l(\mathrm{x})=\|\mathrm{x}\|_A=\sqrt{\mathrm{x}^TA^{-1}\mathrm{x}}, \tag{3.12}$$

where $\mathrm{x}=(x,y)^T$. Notable that,because A is a symmetric positive definite matrix, the relation (3.12) introduces a norm. Now if $\Phi(r)$ is a radial basis function (RBF) based on the classical Euclidean norm, then $\Phi(l)$ is called an anisotropic radial basis function (A-RBF). By a straightforward manipulation [42], it can be easily found that

$$A\nabla\Phi=\frac{1}{l}\Phi'(l)\mathrm{x},$$