Engineering Analysis with Boundary Elements xxx (xxxx) xxx–xxx

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

Boundary element formulation for steady state plane problems in size-dependent thermoelasticity

Arezoo Hajesfandiari, Ali R. Hadjesfandiari, Gary F. Dargush*

Department of Mechanical and Aerospace Engineering, University at Buffalo, State University of New York, Buffalo, NY 14260 USA

ARTICLE INFO

Keywords:
Integral equations
Thermoelastic response
Size-dependence
Skew-symmetric couple-stress
Micromechanics
Nanomechanics

ABSTRACT

A boundary element method is developed to examine two-dimensional size-dependent thermoelastic response in isotropic solids. The formulation is founded on the recently established consistent couple stress theory, in which both the couple-stress tensor and its energy conjugate mean curvature tensor are skew-symmetric. For isotropic materials, there is no thermal mean curvature deformation, and the thermoelastic effect is solely the result of thermal strain deformation. As a result, size-dependency is quantified by one characteristic material length scale parameter l, while the thermal coupling is activated through the classical thermal expansion coefficient α. Interestingly, in this size-dependent multi-physics model, the thermal governing equation is independent of the deformation. However, the mechanical governing equations depend on the temperature field. Here, we develop the boundary integral representation and numerical implementation for this size-dependent thermoelastic boundary element method (BEM) for plane problems, which involves temperatures, displacements, rotations, normal heat fluxes, force-tractions and couple-tractions as primary variables within a boundary-only formulation. Then, we apply this new BEM formulation to several basic computational problems in an effort to validate the robustness of the numerical implementation and to examine size-dependent response.

1. Introduction

Over the last several decades, there has been a continuing thrust to develop technology on progressively smaller length scales. A major challenge is that at reduced scales, the physical behavior of materials becomes size-dependent, which in turn can affect the mechanical, thermal, electrical and magnetic performance of a broad range of components and systems. Although much progress has been made in the development of atomistic methods, continuum approaches become more attractive as one attempts to model over the spatial and temporal scales required for most engineering applications. Consequently, new developments in continuum micromechanics and nanomechanics are needed to capture accurately the behavior of size-dependent phenomena, especially those involving multi-physics response, such as thermomechanics and electromechanics. Here, we focus on thermoelasticity, which pertains to predicting the linear elastic behavior of solids subjected to both thermal and mechanical loadings, and develop a size-dependent boundary element method to solve two-dimensional steady-state linear thermoelastic size-dependent boundary value problems.

Recently, a consistent couple stress theory has been developed in continuum mechanics [1], which can account for the length scale dependency associated with the underlying microstructure of materials. In this theory, the couple-stress is established as a second order skew-symmetric pseudo-tensor, and is energetically conjugate to the skew-symmetric mean curvature pseudo-tensor. This non-classical continuum mechanics theory provides a powerful theoretical framework to develop new size-dependent theories for various coupled multi-physics problems, such as piezoelectricity [2] and thermoelasticity [3]. In the resulting size-dependent thermoelastic theory [3], the temperature can create not only thermal strain deformation, but also thermal mean curvature deformation. However, for isotropic materials, there is no thermal mean curvature deformation, and the thermoelastic effect is solely the result of the thermal strain deformation. This means the temperature rise only creates strain deformation, and the appearance of couple-stresses is the result of a purely mechanical effect. Interestingly, the size-dependency for isotropic materials is specified by a single characteristic length scale parameter l, which makes the theory parsimonious, as well as fully self-consistent.

Although the governing equations for the steady state size-dependent thermoelasticity are linear, analytical solutions remain difficult to obtain. In order for technology to take full advantage of this size-

* Corresponding author.
E-mail address: gdargush@buffalo.edu (G.F. Dargush).

http://dx.doi.org/10.1016/j.enganabound.2017.02.004
Received 26 January 2016; Received in revised form 15 February 2017; Accepted 21 February 2017

dependent phenomena, numerical methods must be developed for accurate modeling and to attain deeper understanding. Consequently, computational mechanics can play an important role in the advancement of micro- and nano-technologies through the solution of a broad range of boundary value problems. The boundary element method is a suitable method, especially in solving the corresponding linear problems. This method is particularly appropriate in cases where accuracy is required due to local phenomena, such as notches, cracks and bimaterial interfaces, or where the domain of interest extends to infinity. The main difficulties in developing boundary element methods are the derivation of the corresponding integral equations, fundamental solutions and the implementation of more complicated numerical algorithms than with other computational approaches, such as the finite element method.

Based upon recent work [1,3], the boundary integral equations have been derived for three-dimensional steady state size-dependent thermoelastic response of isotropic materials, along with the corresponding singular infinite space Green's functions or fundamental solutions [4]. This has been achieved by first developing thermal and mechanical reciprocal theorems for general anisotropic thermoelastic material and then specializing to the isotropic case.

In the present paper, we develop a corresponding two-dimensional boundary element formulation for steady state size-dependent thermoelasticity in isotropic materials. For this formulation, we utilize the general thermal and mechanical reciprocal theorems for size-dependent thermoelasticity [4]. Interestingly, it will be shown that for the two-dimensional case, the kernels are at most strongly singular, which makes possible the implementation of a conventional boundary element method. This present development can be considered as an extension of the boundary element implementation for the two-dimensional couple stress elasticity [5], size-dependent couple stress piezoelectricity [6], and the classical planar thermoelastic formulations developed long ago [7–12]. A computational example based on this boundary element development regarding a cantilever beam with a transverse temperature gradient has been already presented very recently to demonstrate the thermoelastic effects in size-dependent mechanics [13]. We should note that there has been previous work on the elastic and thermoelastic response of materials with size-dependence, based on alternative non-classical theories, such as micropolar theory and second gradient elasticity. For example, boundary element methods in the former category have been developed by Das and Chaudhuri [14], Huang and Liang [15], Liang and Huang [16] and Shmoylova et al. [17]. On the other hand, second gradient elasticity boundary element formulations have been presented by Polyzos et al. [18], Tsepoura et al. [19] and Karlis et al. [20]. However, here we develop the first computational method for fully self-consistent size-dependent thermoelastic theory.

The remaining sections of this paper are organized as follows. A brief overview of steady state size-dependent thermoelasticity for isotropic materials is provided in Section 2, including the definition of the corresponding boundary value problem in the two-dimensional case. In Section 3, the boundary integral representation of this boundary value problem is presented, along with a discussion of the singularities appearing in the two-dimensional kernels. Afterwards, in Section 4, we discuss the numerical implementation of the size-dependent thermoelastic boundary element method, which is then applied in Section 5 to several basic computational examples in order to test the formulation and to study a number of interesting characteristics of size-dependent thermoelastic media. Finally, conclusions and directions for future research are provided in Section 6. Additional detail on the boundary integral equations and fundamental solutions appear in Appendix A.

2. Theoretical overview

In this section, we provide a brief overview of the two-dimensional steady state size-dependent isotropic thermoelasticity theory. For a more detailed discussion on this consistent theory, the reader may wish to consult references on the general development [1], as well as for the thermoelastic case [3].

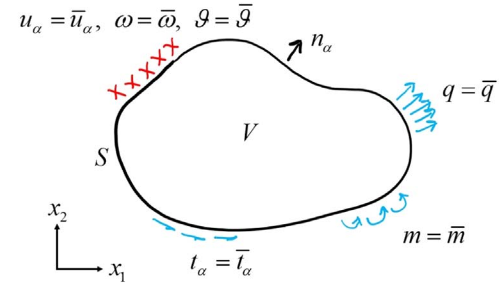

Fig. 1. Size-dependent thermoelastic plane problem definition.

Consider a size-dependent thermoelastic material occupying a cylindrical region, oriented with the x_3-axis parallel to the axis of the cylinder, as illustrated in Fig. 1. Here, V and S represent, respectively, the cross section of the body in the x_1x_2-plane and its bounding edge in that plane. The outward unit normal vector on the boundary surface S is labelled n_α. In this paper, we will use standard indicial notation with Greek indices that vary only over (1,2). Indices after commas denote partial derivatives, parentheses around index pairs define the symmetric part of a second order tensor, while the skew-symmetric part of a second order tensor is identified by using square brackets around the indices.

Assume that the body is undeformed and is stress-free at a uniform absolute reference temperature T_0, when there is no external force and couple. Then, when the body is subjected to external forces and heat sources, it in general may undergo a temperature change (or rise) field $\vartheta = T - T_0$, and an accompanying deformation specified by the displacement field u_α. These result in heat conduction and internal stresses in the body. For the problems under consideration here, all quantities are independent of x_3.

The fundamental thermomechanical equations for steady state size-dependent thermoelastic theory are the energy balance law, generalized force and moment equilibrium equations [3]. For the plane problem, these governing equations reduce to the following:

Energy balance equation

$$-q_{\alpha,\alpha} + Q = 0 \tag{1a}$$

Force equilibrium equation

$$\sigma_{\beta\alpha,\beta} + F_\alpha = 0 \tag{1b}$$

Moment equilibrium equation

$$\sigma_{[\beta\alpha]} = -\mu_{[\alpha,\beta]} \tag{1c}$$

where q_α represents the heat flux vector, $\sigma_{\alpha\beta}$ and μ_α are the force-stress tensor and couple-stress vector, respectively, Q is the quantity of heat generated per unit time and volume in the body and F_α is the body force per unit volume.

Notice that the moment equilibrium Eq. (1c) provides the relationship between the following non-zero components of force-stress and couple-stress:

$$\sigma_{[21]} = -\sigma_{[12]} = -\mu_{[1,2]} \tag{2}$$

As a result, the force equilibrium Eq. (1b) reduce to

$$\left[\sigma_{(\beta\alpha)} - \mu_{[\alpha,\beta]}\right]_{,\beta} + F_\alpha = 0 \tag{3}$$

For kinematics of the size-dependent continuum mechanics problem, the out-of-plane rotation component can be written as