Engineering Analysis with Boundary Elements 84 (2017) 19–24

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

The frequency averaged normal-derivative integral equation to predict frequency averaged quadratic pressure radiated from structures

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ARTICLE INFO

Keywords:
FAQP
Acoustic radiation
High frequency
Irregular frequency
Normal-derivative integral equation
Burton and Miller formulation

ABSTRACT

Frequency averaged normal-derivative Helmholtz integral equation is proposed to get robust predictions of the frequency averaged quadratic pressure (FAQP) radiated from the structures at medium and high frequencies. The non-uniqueness problem of frequency averaged Helmholtz integral equation and frequency averaged normal-derivative Helmholtz integral equation is overcome by the coupling method combining these two integral equations. The numerical examples are given to demonstrate the versatility of the frequency averaged normal-derivative Helmholtz integral equation and the coupling method.

1. Introduction

Conventional boundary element method (conventional BEM) has been widely applied in acoustics for the solution of radiation and scattering boundary value problems [1]. However, the prediction of noise radiated from machinery with conventional BEM is a difficult problem for four main reasons: firstly, sufficient elements are dispersed to predict the pressure radiated from structures for accuracy. It means that more elements are needed to resolve both the vibration and sound pressure at higher frequencies. Secondly, for the real-life structures with a very sensitive vibro-acoustic behavior, the radiating pressure strongly varies at a given frequency for different objects of the identical industrial production due to small dispersion of geometry, material and manufacturing process. Thirdly, frequency averaging manipulation of each pure tone prediction at medium and high frequencies makes conventional BEM troublesome in the process of predicting noise radiated from structures submitted to broad band excitation. Fourthly, it is impossible to describe the vibrational behavior of large and complex machines with enough information about the field-measured vibration with great changes, especially at high frequencies, even if the same measurements are repeated [2–5].

For enclosed high-frequency sound fields, a boundary element method was developed by Franzoni [6–8] in terms of time-averaged energy and intensity variables with broadband acoustic energy/intensity sources. In order to get robust predictions at medium and high frequencies in an unbounded medium, a general integral equation approach to predict the Frequency Averaged Quadratic Pressure (FAQP) radiated from vibrating bodies was proposed by Guyader in [3]. As mentioned in [3], the frequency averaging does not suppress the characteristic wavenumber problem and regularization methods have to be used in a similar way to avoid the calculation of the instability. In the conventional BEM, to overcome the non-uniqueness problem, two major formulations, the Combined Helmholtz Integral Equation Formulation (CHIEF) and the Burton and Miller method, were proposed by Schenck [9] and by Burton and Miller [10], respectively. The CHIEF method generates an overdetermined system of equations to find the solution that satisfies both the surface and the interior Helmholtz integral equation to predict pressure in the exterior. The CHIEF method's primary shortcoming is that for an arbitrary geometry, the nodal surfaces are not known a priori, so the selection of number and location of the CHIEF points becomes critical [11], especially at high frequencies, although Wu improved the CHIEF method by using a few CHIEF points at every frequency and increase the number of CHIEF points as frequency increases [12]. The Burton and Miller method provides unique solutions for all frequencies by using a linear combination of the Helmholtz integral equation and its normal derivative integral equation. Although the major drawback of this method is that normal-derivative Helmholtz integral equation has a hypersingular kernel of $1/r^3$, the hypersingular integral has been solved by several literatures [13–20]. To overcome the non-uniqueness problem of FAQP method, a combined energy boundary integral equation formulation (CEBIEF) [21] was proposed to obtain unique solutions at irregular frequencies. However, the CEBIEF method

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http://dx.doi.org/10.1016/j.enganabound.2017.07.016
Received 1 December 2016; Received in revised form 22 June 2017; Accepted 17 July 2017
0955-7997/

has the similar disadvantages of the CHIEF method such as the problem about the selection of number and location of the CHIEF points.

The goal of this paper is to derive a frequency averaged normal-derivative integral equation to predict frequency averaged quadratic pressure radiated from structures and propose a coupling method to solve the non-uniqueness problem.

2. Theoretical formulation

2.1. Integral equation for frequency averaged quadratic pressure [3]

The frequency averaged quadratic pressure radiated at point P of the acoustic medium can be calculated from Eq. (1)

$$\begin{aligned}\langle|p(P)|^2\rangle &= \int_S\int_S \langle\rho^2k^2c^2G(Q,P)G^*(Q',P)\rangle S_{uu}(Q,Q')dS(Q)dS(Q') \\ &+ \int_S\int_S\left\langle\frac{\partial}{\partial n}G(Q,P)\frac{\partial}{\partial n}G^*(Q',P)\right\rangle S_{pp}(Q,Q')dS(Q)dS(Q') \\ &+2Re\left(\int_S\int_S\left\langle i\rho kcG(Q,P)\frac{\partial}{\partial n}G^*(Q',P)\right\rangle S_{up}(Q,Q')dS(Q)dS(Q')\right),\end{aligned} \tag{1}$$

where the function

$$G(Q,P)=e^{-ikR}/4\pi R \tag{2}$$

is the free-space Green's function in which $R=|Q-P|$, the collocation point P is in the acoustic domain. The point Q and Q' are located on the surface and the superscript '*' denotes the complex conjugate. n is the normal vector at Q on S. $i=\sqrt{-1}$. k is the wavenumber, c is the sound speed and ρ is the density of the acoustic medium. The expressions of the boundary sources are given by Eqs. (3)–(6), respectively.

$$S_{uu}(Q,Q')=\langle V_n(Q)V_n^*(Q')\rangle, \tag{3}$$

$$S_{pp}(Q,Q')=\langle P(Q)P^*(Q')\rangle, \tag{4}$$

$$S_{up}(Q,Q')=\langle V_n(Q)P^*(Q')\rangle, \tag{5}$$

$$S_{pu}(Q,Q')=\langle P(Q)V_n^*(Q')\rangle. \tag{6}$$

The frequency average is made on a wavenumber bandwidth $2\Delta k$ centered at Ω by Eq. (7)

$$\langle\ \rangle=\frac{1}{2\Delta k}\int_{\Omega-\Delta k}^{\Omega+\Delta k}dk. \tag{7}$$

The frequency averaged sound power can be calculated by

$$\langle\text{power}\rangle=\sum_i^N\frac{1}{2}\text{Re}(S_{up}(i,i))S_i, \tag{8}$$

where S_i represents the area of the ith element and N denotes the total number of elements.

2.2. The frequency averaged Helmholtz integral equation [3]

The boundary sources can be determined by the integral equations of frequency averaged Helmholtz integral equation, Eqs. (9) and (10) [3,15,21]

$$\begin{aligned}\frac{1}{2}S_{pu}(L,L') &= \int_{S-\Delta_Q}\langle -i\rho kcG(Q,L)\rangle S_{uu}(Q,L')dS(Q) \\ &-\int_S\left\langle\frac{\partial}{\partial n}G(Q,L)\right\rangle S_{pu}(Q,L')dS(Q),\end{aligned} \tag{9}$$

$$\begin{aligned}\frac{1}{2}S_{pp}(L,L') &= \int_{S-\Delta_Q}\langle -i\rho kcG(Q,L)\rangle S_{up}(Q,L')dS(Q) \\ &-\int_S\left\langle\frac{\partial}{\partial n}G(Q,L)\right\rangle S_{pp}(Q,L')dS(Q),\end{aligned} \tag{10}$$

where $S-\Delta_Q$ denotes the boundary S excluding the boundary elements Δ_Q in which the point Q and collocation point L is located. The point L' is located on the surface S.

2.3. The frequency averaged normal-derivative Helmholtz integral equation

With the normal-derivative Helmholtz integral equation [22] multiplied by $V_n^*(L')$ and $P^*(L')$, the products write respectively.

$$\begin{aligned}-\frac{1}{2}i\rho kcV_n(L)V_n^*(L') &= \int_{S-\Delta_Q}-i\rho kc\frac{\partial G(Q,L)}{\partial n^L}V_n(Q)V_n^*(L')dS(Q) \\ &-\int_S\frac{\partial^2G(Q,L)}{\partial n\partial n^L}p(Q)V_n^*(L')dS(Q),\end{aligned} \tag{11}$$

$$\begin{aligned}-\frac{1}{2}i\rho kcV_n(L)P^*(L') &= \int_{S-\Delta_Q}-i\rho kc\frac{\partial G(Q,L)}{\partial n^L}V_n(Q)P^*(L')dS(Q) \\ &-\int_S\frac{\partial^2G(Q,L)}{\partial n\partial n^L}P(Q)P^*(L')dS(Q),\end{aligned} \tag{12}$$

where n^L is the normal vector at point Lon S.

With the frequency averaging (Eq. (7)) of Eqs. (11) and (12) and the assumption [3], the statistical independence of boundary pressure and velocity and of the Green's functions, Eqs. (13) and (14) are derived, respectively.

$$\begin{aligned}\left\langle-\frac{1}{2}i\rho kc\right\rangle S_{uu}(L,L') &= \int_{S-\Delta_Q}\left\langle-i\rho kc\frac{\partial G(Q,L)}{\partial n^L}\right\rangle S_{uu}(Q,L')dS(Q) \\ &-\int_S\left\langle\frac{\partial^2G(Q,L)}{\partial n\partial n^L}\right\rangle S_{pu}(Q,L')dS(Q),\end{aligned} \tag{13}$$

$$\begin{aligned}\left\langle-\frac{1}{2}i\rho kc\right\rangle S_{up}(L,L') &= \int_{S-\Delta_Q}\left\langle-i\rho kc\frac{\partial G(Q,L)}{\partial n^L}\right\rangle S_{up}(Q,L')dS(Q) \\ &-\int_S\left\langle\frac{\partial^2G(Q,L)}{\partial n\partial n^L}\right\rangle S_{pp}(Q,L')dS(Q).\end{aligned} \tag{14}$$

The boundary sources for Eq. (1) can be determined by Eqs. (13) and (14).

2.4. The coupling method to overcome the non-uniqueness problem

Although one can use CEBIEF method [21] to get a unique solution, it also accompanies the difficulty that the selection of number and location of the interior points becomes critical in consideration of that the nodal surfaces are not known a priori for an arbitrary geometry. The coupling method is proposed to robustly overcome the non-uniqueness of FAQP method at irregular frequencies for an arbitrary geometry by combining the frequency averaged Helmholtz integral equation with frequency averaged normal-derivative Helmholtz integral equation.

$$\begin{aligned}&\frac{1}{2}S_{pu}(L,L')+\int_{S-\Delta_Q}\left\langle\frac{\partial}{\partial n}G(Q,L)\right\rangle S_{pu}(Q,L')dS(Q) \\ &+\alpha\left(-\int_S\left\langle\frac{\partial^2G(Q,L)}{\partial n\partial n^L}\right\rangle S_{pu}(Q,L')dS(Q)\right) \\ &=\int_S\langle -i\rho kcG(Q,L)\rangle S_{uu}(Q,L')dS(Q)+\alpha\left(\left\langle-\frac{1}{2}i\rho kc\right\rangle S_{uu}(L,L')\right. \\ &\left.-\int_{S-\Delta_Q}\left\langle-i\rho kc\frac{\partial G(Q,L)}{\partial n^L}\right\rangle S_{uu}(Q,L')dS(Q)\right),\end{aligned} \tag{15}$$

$$\begin{aligned}&\frac{1}{2}S_{pp}(L,L')+\int_{S-\Delta_Q}\left\langle\frac{\partial}{\partial n}G(Q,L)\right\rangle S_{pp}(Q,L')dS(Q) \\ &+\alpha\left(-\int_S\left\langle\frac{\partial^2G(Q,L)}{\partial n\partial n^L}\right\rangle S_{pp}(Q,L')dS(Q)\right) \\ &=\int_S\langle -i\rho kcG(Q,L)\rangle S_{up}(Q,L')dS(Q)+\alpha\left(\left\langle-\frac{1}{2}i\rho kc\right\rangle S_{up}(L,L')\right. \\ &\left.-\int_{S-\Delta_Q}\left\langle-i\rho kc\frac{\partial G(Q,L)}{\partial n^L}\right\rangle S_{up}(Q,L')dS(Q)\right),\end{aligned} \tag{16}$$