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Improved complex variable moving least squares approximation for three-dimensional problems using boundary integral equations

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ABSTRACT

The improved complex variable moving least squares approximation is an efficient method to generate meshless approximation functions. In the past, the approximation has been used only for 2D problems. In this paper, the approximation is developed to solve 3D problems. Theoretical error estimation of the approximation is given. Then, incorporating the approximation into boundary integral equations, a symmetric and boundary-only meshless method, the complex variable Galerkin boundary node method, is developed and analyzed theoretically for 3D potential, Helmholtz and Stokes problems. Numerical results demonstrate the accuracy and efficiency of the developed method.

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1. Introduction

Meshless (or meshfree) methods have been developed quickly in the past two decades to deal with the meshing-related disadvantages involved in some traditional numerical methods such as the finite element method and the boundary element method. In meshless methods, approximations of variables need only nodes instead of elements. Thus, how to approximate variables is very important in meshless methods. The moving least squares (MLS) approximation [1] is one of the main methods to approximate variables in meshless methods. By using the MLS approximation, some meshless methods, such as the element-free Galerkin (EFG) method [2,3], the meshless local Petrov–Galerkin (MLPG) method [4] and the boundary node method (BNM) [5] have been developed for numerical analysis of many boundary value problems. The MLS approximation can produce approximation functions with high smoothness and precision. Nevertheless, because it is the approximation of scalar functions, the associated meshless methods require many nodes in the computational domain [6–8].

To conquer the shortcoming of the MLS approximation, a complex variable moving least squares (CVMLS) approximation [6,7] has been developed by incorporating the complex variable theory into the MLS approximation. In the CVMLS approximation, 2D approximation function is obtained by using 1D basis function. Hence, fewer coefficients and fewer nodes are needed in the CVMLS approximation than in the MLS approximation. The CVMLS-based meshless methods have higher computational accuracy and consume less CPU times than the associated MLS-based meshless methods [7–11]. As in the MLS approximation, the functional used in the CVMLS approximation to solve the unknown coefficients is defined directly in a weighted squares form. Since the approximated function is expressed as a complex variable function, the value of the functional is a complex number. Recently, an improved complex variable moving least squares (ICVMLS) approximation [12,13] has been developed by using the complex modulus to define a new functional. The ICVMLS approximation inherits all merits of the CVMLS approximation. Besides, the new functional denotes the best approximation and its value is a real number. The ICVMLS-based meshless methods have higher computational efficiency and precision than the CVMLS-based methods [12–16].

Since the complex variable theory is valid and perfect for 2D space, complex variable meshless methods have been applied successfully for the numerical solution of 2D boundary value problems. However, the application of these complex variable meshless methods to 3D problems has not been found until now. It is therefore important to develop complex variable meshless methods for the solution of 3D problems.

This paper is devoted to the development of the ICVMLS approximation for solving 3D boundary value problems. The Galerkin boundary node method (GBNM) [17,18] is a meshless method that combines the MLS approximation with boundary integral equations (BIEs) [19,20]. In the GBNM, boundary conditions are implemented with ease, and the resulting system matrix is symmetric and positive definite. In this paper, the ICVMLS approximation is introduced into the GBNM to produce a complex variable Galerkin boundary node method (CVGBNM)

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for numerical analysis of 3D potential, Helmholtz and Stokes problems. With the aid of BIEs, the ICVMLS approximation used in the CVGBNM is performed only on the surface of a 3D domain. Using curvilinear coordinates, the 3D surface can be expressed by two parameters (s_1, s_2). Then, formulations of the ICVMLS approximation on the surface of a 3D domain are developed by 1D basis functions. Owing to the merits of the ICVMLS approximation and the GBNM, the proposed CVGBNM is expected to possess higher computational efficiency. Theoretical error estimation of the ICVMLS approximation for 3D functions is given. Then, error of the CVGBNM for 3D potential, Helmholtz and Stokes problems is also analyzed.

The rest of this paper is outlined as follows. In Section 2, the ICVMLS approximation on a 3D surface is discussed, and error of the ICVMLS approximation is also presented. Then, analysis of the CVGBNM for 3D potential, Helmholtz and Stokes problems is given in Sections 3 and 4. Finally, numerical results and conclusions are provided in Sections 5 and 6, respectively.

2. The ICVMLS approximation on 3D surfaces

In this section, the ICVMLS approximation is developed and analyzed on 3D surfaces. Because only boundary nodes are required in the CVGBNM for approximation of boundary unknowns, the ICVMLS approximation is only carried out on the 3D bounding surface.

2.1. Notations

Let Ω be a bounded domain in $\mathbb{R}^3$ with boundary Γ and let Ω' be the complementary of $\bar{\Omega} = \Omega \cup \Gamma$, that is the exterior of $\bar{\Omega}$. A general point of $\mathbb{R}^3$ is denoted by $\mathbf{x} = (x_1, x_2, x_3)^{\mathrm{T}}$. Assume that the bounding surface Γ is the union of piecewise smooth segments called panels, and the edges of these panels are Lipschitz continuous. On each panel, surface curvilinear coordinates are defined as $\mathbf{s} = (s_1, s_2)^{\mathrm{T}}$. Then, Γ can be represented in parametric forms as $x_j = x_j(s_1, s_2)$ with $j = 1, 2, 3$. For any point $\mathbf{x} = \mathbf{x}(s_1, s_2) \in \Gamma$, let $z = s_1 + \mathrm{i}s_2$, where $\mathrm{i} = \sqrt{-1}$ is an imaginary number. Assume that the influence domain of z is

$$\mathfrak{R}(z) = \{\tilde{z} \in \Gamma : |\tilde{z} - z| \le h(z)\}$$

Let $\{\mathbf{x}_I\}_{I=1}^N$ be a set of N nodes on Γ. These boundary nodes can also be denoted as $\{z_I\}_{I=1}^N$. Let $\wedge(z) \overset{\Delta}{=} \{I_1, I_2, \dots, I_\tau\} \subseteq \{1, 2, \dots, N\}$ be the set of the global sequence numbers of boundary nodes whose influence domains cover the point z, and let

$$w_I(z) = \varphi\left(\frac{|z - z_I|}{h_I}\right), \qquad I = 1, 2, \dots, N \tag{1}$$

be weight functions. Here, the function φ is nonnegative, γ-th times continuously differentiable, and its derivatives up to order γ are bounded. The weight functions $w_I(z)$ also have compact supports

$$\mathfrak{R}_I \overset{\Delta}{=} \mathfrak{R}(z_I) = \{z \in \Gamma : |z - z_I| \le h_I\} \tag{2}$$

where h_I is the radius of $\mathfrak{R}_I$. Then, we have

$$I \in \wedge(z) \Leftrightarrow z \in \mathfrak{R}_I \Leftrightarrow w_I(z) > 0, \qquad \forall z \in \Gamma, \qquad I = 1, 2, \dots, N$$

Let

$$h = \max_{1 \le I \le N} \min_{1 \le J \le N, J \ne I} |z_I - z_J|$$

be the nodal spacing. In this paper, the letter C will denote a general constant which is independent of h and is not necessarily the same in each occurrence. For theoretical analysis, assuming the data site $\{z_I\}_{I=1}^N$ satisfies the following quasi-uniform condition [9,10,21]

$$C_1 h \le h_I \le C_2 h, \qquad I = 1, 2, \dots, N \tag{3}$$

where C_1 and C_2 are two positive constants independent of h.

For functions f and g, we define the inner product about $\{z_I\}_{I \in \wedge(z)}$ as $(f, g)_z = \sum_{I \in \wedge(z)} w_I(z) f(z_I) \bar{g}(z_I)$, where $\bar{g}(z_I)$ is the conjugate of $g(z_I)$. The corresponding norm is $\|f\|_z = \sqrt{(f, f)_z}$.

2.2. Formulations

Let u_1 and u_2 be two real value functions defined on Γ. In the ICVMLS approximation, the local approximation of $u(z) = u_1(z) + \mathrm{i}u_2(z)$ can be defined as

$$u(z) \approx \mathcal{M}u(z, z^*) = \bar{\mathbf{p}}^{\mathrm{T}}(z^*)\mathbf{a}(z), \qquad \forall z \in \Gamma \tag{4}$$

where $\mathcal{M}$ is an approximation operator, the point z^* can either be the evaluation point z or a node $z_I \in \mathfrak{R}(z)$, $\mathbf{a}(z)$ is a $(m+1)$-dimensional column vector, and

$$\bar{\mathbf{p}}^{\mathrm{T}}(z) = [\bar{p}_0(z), \bar{p}_1(z), \bar{p}_2(z), \dots, \bar{p}_m(z)]$$

is the conjugate of the basis vector

$$\mathbf{p}^{\mathrm{T}}(z) = [p_0(z), p_1(z), p_2(z), \dots, p_m(z)]$$

The basis vector can be chosen as [7,8,13] $\mathbf{p}^{\mathrm{T}}(z) = [1, z, z^2, \dots, z^m]$. However, as in the MLS approximation [21,22], to obtain more stable results, we should use the scaled and shifted basis vector [9,10],

$$\mathbf{p}^{\mathrm{T}}(z) = \left[1, \frac{z - z^e}{h}, \left(\frac{z - z^e}{h}\right)^2, \dots, \left(\frac{z - z^e}{h}\right)^m\right] \tag{5}$$

where z^e is fixed and can be chosen as the evaluation point z in practical computations.

The unknown vector $\mathbf{a}(z)$ in Eq. (4) is deduced by minimizing the following functional

$$\mathcal{J}(z) = \sum_{I \in \wedge(z)} w_I(z)\left|\bar{\mathbf{p}}^{\mathrm{T}}(z_I)\mathbf{a}(z) - u(z_I)\right|^2 = \left\|\bar{\mathbf{p}}^{\mathrm{T}}\mathbf{a}(z) - u\right\|_z^2$$

We can obtain the minimum of $\mathcal{J}$ by choosing $\mathbf{a}(z)$ such that $\bar{\mathbf{p}}^{\mathrm{T}}\mathbf{a}(z)$ is the projection of u in the space $\mathrm{span}\{\bar{p}_0, \bar{p}_1, \dots, \bar{p}_m\}$. Then,

$$\left(\bar{\mathbf{p}}^{\mathrm{T}}\mathbf{a}(z), \bar{p}_j\right)_z = \left(u, \bar{p}_j\right)_z, \qquad j = 0, 1, \dots, m$$

i.e.,

$$\left[\sum_{I \in \wedge(z)} w_I(z)\bar{\mathbf{p}}^{\mathrm{T}}(z_I)p_j(z_I)\right]\mathbf{a}(z) = \sum_{I \in \wedge(z)} w_I(z)u(z_I)p_j(z_I),$$
$$j = 0, 1, \dots, m$$

It is expressed in matrix form as

$$\mathbf{A}(z)\mathbf{a}(z) = \mathbf{B}(z)\mathbf{u} \tag{6}$$

where $\mathbf{u} = [u(z_{I_1}), u(z_{I_2}), \dots, u(z_{I_\tau})]^{\mathrm{T}}$, and the entries of $\mathbf{A}(z)$ and $\mathbf{B}(z)$ are

$$\mathbf{A}_{kj}(z) = \sum_{I \in \wedge(z)} w_I(z)\bar{p}_k(z_I)p_j(z_I), \qquad k, j = 0, 1, \dots, m \tag{7}$$

and

$$\mathbf{B}_{j\ell}(z) = w_{I_\ell}(z)p_j(z_{I_\ell}), \qquad j = 0, 1, \dots, m; \qquad \ell = 1, 2, \dots \tau; \qquad I_\ell \in \wedge(z) \tag{8}$$

respectively.

Solving $\mathbf{a}(z)$ from Eq. (6) and substituting it into Eq. (4), the ICVMLS approximation is finally expressed as

$$u(z) \approx \mathcal{M}u(z) = \mathcal{M}u(z, z^*)\big|_{z^*=z} = \bar{\mathbf{p}}^{\mathrm{T}}(z)\mathbf{A}^{-1}(z)\mathbf{B}(z)\mathbf{u} = \sum_{I=1}^N \Phi_I(z)u_I \tag{9}$$

where the ICVMLS shape functions are

$$\Phi_I(z) = \begin{cases} \sum_{j=0}^m \bar{p}_j(z)\left[\mathbf{A}^{-1}(z)\mathbf{B}(z)\right]_{jk}, & I = I_k \in \wedge(z), \\ 0, & I \notin \wedge(z), \end{cases} \qquad I = 1, 2, \dots, N \tag{10}$$

In Eq. (9), the ICVMLS approximation is shown for the complex value function $u = u_1 + \mathrm{i}u_2$. When u is a real value function, by setting $u_1 = u$ and $u_2 = 0$, the ICVMLS approximation of u is [8]

$$u(z) \approx \mathcal{M}u(z) = \sum_{I=1}^N \left[\mathrm{Re}\Phi_I(z)\right]u(z_I) \tag{11}$$

where $\mathrm{Re}\Phi_I(z)$ denotes the real part of $\Phi_I(z)$.

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